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Evaluation method for the conventional oedometer test

Une méthode d'évaluation pour le test oedomètre

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ABSTRACT: The conventional oedometer test is approximately evaluated (with one-point fit) in practice using the primary consolidation model of Terzaghi. The aim of the research is to validate a model for both staged tests, and to elaborate a more precise routine evaluation method using the more complex models, an automatic inverse solver and some reliability testing methods. The model used consisted of a consolidation part-model and a creep part-model and a term for the immediate compression, the mathematically accurate inverse problem solver is automatic. The evaluation of some compression tests showed that the fitting error decreased to the half if the suggested model was used. The initial compression parameter was likely controlled by the compressibility of the grains, being negligible for silts, non-negligible for clays and peat. The reliability of the solution of the model fitting was not ensured unless this parameter was added in case of waste. The suggested approach can routinely be used in geotechnical labs (a software package is available) with the advantage that by separating the primary consolidation, the immediate compression and the creep settlements, a more precise routine settlement analysis can be made on the true scale as well.

RÉSUMÉ: Le test oedométrique conventionnel est approximativement évalué (en utilisant un ajustement en un point) en pratique avec le modèle de consolidation primaire de Terzaghi. L'objectif de la recherche est d'élaborer une méthode d'évaluation de routine plus précise utilisant un modèle légèrement plus complexe, un résolveur inverse automatique et certaines méthodes de test de fiabilité. Le modèle utilisé consistait en un modèle partiel de consolidation et un modèle partiel de fluage et un terme pour la compression immédiate, le résolveur de problèmes inverse mathématiquement précis est automatique. L'évaluation de certains essais de compression effectués a montré que l'erreur d'ajustement diminuait de moitié si le modèle proposé était utilisé. Le paramètre de compression initial était contrôlé par la compressibilité des grains, étant négligeable pour les limons, non négligeable pour les argiles et la tourbe. La fiabilité de la solution de l'ajustement du modèle n'était assurée que si ce paramètre était ajouté en cas de déchets. L'approche suggérée peut être couramment utilisée dans les laboratoires géotechniques (un progiciel est disponible), avec l'avantage qu'en séparant la consolidation primaire, la compression immédiate et les règlements de fluage, une analyse de règlement de routine plus précise peut être réalisée à l'échelle réelle.+.

Keywords: compression test, creep, relaxation, model fittig, model validation

1 INTRODUCTION

1.1 Staged laboratory tests

The empirical equation of creep and relaxation can be determined with the staged oedometer tests if the stage duration is longer than 99% of the consolidation time. The creep can be observed during the conventional oedometric compression test (OCT) with constant total stress load, and the relaxation can be observed with the new oedometric relaxation test (ORT), where constant displacement load is applied. These dual tests are some species of the one-dimensional oedometric test (Lerouelli et al, 1985) that can be used to determine the compression curve, the coefficient of consolidation (or the permeability), and the viscous properties of the saturated soils. No validated model is available for the staged tests.

1.2 Evaluation of staged laboratory tests

In routine applications the compression test is evaluated with Terzaghi's model in an approximate way, as follows:

$$c = \frac{T_{\kappa}}{t_k} R \quad (1)$$

where R is a parameter depending on the initial condition which is the constant function, T_{κ} is a theoretical time factor, and t_{κ} is the measured time at the κ degrees of dissipation.

The first problem is that the theoretical dissipation curves are given as a function of the initial condition, which is assumed to be uniform, this assumption is not met in the case of partly drained load. The second problem is that the inverse problem solution is approximate. The potential error of this method is uncertain and may be as great as one order of magnitude and this error is not routinely estimated.

The third problem is that as the accuracy improves with increasing duration of the measurement, the measured data are generally related to 90% of the dissipation time, not too many stages are applied and the precise shape of the compression curve is not explored in practice.

1.3 Aim and content of the paper

The objective of this study is to validate a model and then elaborate an automatic and precise evaluaton method for both staged tests so that the measured data can be evaluated mathematically precisely and the testing time reduced. A short multistage testing procedure for the two staged oedometric tests (with shorter stages than 99% of the dissipation time except for the last stage) is suggested.

For these objectives, laboratory tests were made on identical soil samples with different plasticity,that included some short multistage oedometric relaxation tests (MRTs) and some standard and short multistage oedometric compression tests (MCTs). A general inverse problem solution method was suggested and a model validation - discrimination was performed using the variants of the suggested models.

2 LABORATORY TESTS

More than 30 identical sample pairs were tested with the suggested quick MRT and standard and short MCT procedures.

A brief summary is given of the laboratory tests results, only 10 soils are discussed here (Table 1).

2.1 Soils

The soils were quasi-saturated clay soils sampled from Szeged and Szolnok area (Tisza-Körös plane, [5], [6]). The clays were either from below the crust being slightly overconsolidated (OC) or were 'sometimes crust soils' with slightly larger OCR (e.g., sample 5). A highly OC natural bentonite clay was also tested from the Hungarian Central Mountains. Undisturbed samples were used The suction of the quasi-saturated undisturbed soils was released before the tests.

2.2 Procedure

For the MCT, the conventional double-drained multistage compression test equipment was used where the load imposition was instantaneous and was doubled for the standard tests. In addition, the short multistage procedure was applied with 10 to 30 min long stages on some identical sample pairs.

For the MRT, the Geonor type automatic swelling pressure apparatus h-200 A was employed in conjunction with a data acquisition system.

A short multistage procedure was applied with 10 to 20 min long stages, only the last stage was longer than the 99% consolidation time.

The displacement load was increased in equal steps of 0.1 mm. The load imposition was made by about a constant mechanical power. The strain rate decreased hyperbolically with the staged serial number varying between 0.002 to 0.08 %/s. Being larger than 0.001 %/s, this rate of strain is 'too great' in terms of the constitutive relation of Leroueli et al, 1985.

The pore water pressure at the bottom, the total stress, and the displacement at the top of the sample were measured. The displacement was controlled by a microprocessor. The displacement was kept in a tolerance range. As a results, some partial unloading was caused in the sample before the stages (Imre et al, 2015).

3 MODELS

3.1 Assumptions

It is assumed that the total normal stress during a stage of the relaxation test can be described by a joined model consisting of a linear coupled consolidation part-model (for saturated soils) and a relaxation part-model which are superimposed (Imre, 1995 to 1998):

$$\sigma(t) = \sigma^c(t) + \Delta\sigma^r(t) \quad (2)$$

where superscripts c and r indicate the consolidation and, the relaxation part-models, respectively. The relaxation term – including partial unloading – can be formulated as follows.

$$\Delta\sigma\ (t) = s\sigma(0)\frac{1}{1-sb}\log\frac{t}{t_0+t_3} \xrightarrow{t_3=0} \Delta\sigma(t) = -s\cdot\sigma(0)\cdot\log\frac{t}{t_0} \quad (3)$$

where σ is the total normal stress, s is the coefficient of relaxation, t_0 is a technically necessary parameter, t_3 is the pause of relaxation, and $b=\log((t_1+t_3)/t_1)$ is the total stress correction due to partial unloading. This reduces to monotonic variant of the model if t_3 is zero.

It is assumed that the displacement v during a stage of the compression test can be described by a joined model consisting of a linear coupled consolidation part-model (for saturated soils) and a creep part-model amd an immediate compression term which are superimposed:

$$v(t,y) = v^i(t,y) + v^c(t,y) + \varepsilon^{cr}{}_{mean}(t)(H-y) \quad (4$$

where superscripts i indicates immediate component, c and cr indicate consolidation and creep part-models, respectively. The creep part-model after monotonic loading:

$$\Delta e(t) = C_\alpha \log\frac{t}{t_0} \quad (5)$$

where C_α is the coefficient of creep, t_0 is a technically necessary time parameter. The creep term is zero if $t \leq t_0$.

Table 1. Soil physical parameters – plastic soils

	z [m]	I_P %	w_L [%]	OCR	e [-]
1	14	17.0	41.7	1.05	0.66
2	17	22.8	57.9	1.05	0.69
3	11	29.9	63.0	1.05	0.98
4	14	31.7	56.1	1.05	0.75
5	3	37.0	64.1	3.60	0.76
6	11	37.8	63.6	1.05	0.77
7	14	41.0	72.0	1.05	0.83
8	1	62.8	118.7	4.35	1.80

Initial condition

It is assumed that in the case of instantaneous load imposition the Terzaghi's uniform initial pore water pressure distribution is applicable. In any other case the initial pore water pressure distribution can be identified using the following two parametric shape functions. Function E':

$$u(0,y) = G\left(1 - e^{-\frac{y}{F}}\right) \quad F \neq 0, \pm\infty; \quad 0 \leq y \leq H \tag{6}$$

where F, G are parameters. The 'function H':

$$u(0,y) = Ay^3 + By^2 + Cy \tag{7}$$

where A, B and C are parameters, H is the Terzaghi's sample size. The initial condition was characterized by the mean coordinate D of the normalized shape functions which was defined as follows:

$$D = \frac{1}{u(0,H)H} \int_0^H u(0,y)\, dy \tag{8}$$

3.2 The consolidation model

3.2.1 System of differential equations

The 'Equilibrium Equation' compiles the equilibrium condition, the effective stress equality, the geometrical and, the constitutive equations, as follows:

$$E_{oed} \frac{\partial \varepsilon}{\partial y} - \frac{\partial u}{\partial y} = 0 \tag{9}$$

and, the 'Continuity Equation' compiles the continuity equation, the Darcy's law and the geometrical equation, as follows:

$$-\frac{k}{\gamma_v} \Delta u + \frac{\partial \varepsilon}{\partial t} = 0 \tag{10}$$

where v is displacement, u is pore water pressure (neglecting the gravitational component of the hydraulic head), y and t are space and time co-ordinates respectively, E_{oed} is the oedometric modulus:

$$E_{oed} = \frac{2G(1-\mu)}{1-2\mu} = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}, \mu < 0.5 \tag{11}$$

G is the shear modulus, E is the is Young modulus, μ is the Poisson's ratio in terms of the effective stress σ' ($\sigma' = \sigma - u$ where σ is the total normal stress), k is the permeability, γ_V is the unit weight of water, ε is the volumetric strain.

3.2.2 Boundary conditions

In the following four boundary conditions are presented. Three are common, one is different for the two models.

(1) The (common) *boundary condition Nr. 1*:

$$u(t,r)|_{y=0} = 0 \tag{12}$$

(2) The (common) *boundary condition Nr. 2*:

$$\frac{\partial u(t,y)}{\partial y}\Big|_{y=H} \equiv 0 \tag{13}$$

(3) The (common) *boundary condition Nr. 3*:

$$v(t,y)|_{\equiv y=H} \equiv 0 \tag{14}$$

(4) Boundary condition Nr. 4 concerning the coupled 1 models:

$$v(t,y)|_{y=0} \equiv v_0 > 0 \tag{15}$$

(5) *Boundary condition Nr. 5* concerning the compression test models:

$$\varepsilon(t,r)|_{y=0} \equiv \varepsilon_0 > 0. \tag{16}$$

3.2.3 Analytical solution

Steady-state solution part

The solution for the relaxation test model:

$$v^p(y) = v_0\left(1 - \frac{y}{H}\right) \tag{17}$$

$$\sigma^p(y) = -\frac{E_{oed} v_0}{H} \qquad (18)$$

and, for the compression test model:

$$v^p(y) = \frac{\sigma_0}{E_{oed}}(H - y). \qquad (19)$$

$$\sigma^p(y) = \sigma_0 \qquad (20)$$

Transient solution part

The displacement v^t for the oedometric relaxation test model:

$$v^t(t,y) = \sum_{k=1}^{\infty} a_k \cdot \sin\left(\frac{k \cdot \pi}{H} y\right) \cdot e^{-\frac{k^2 \cdot \pi^2}{H^2} \cdot c_v \cdot t} \qquad (21)$$

where a_k (k=1...∞) are the Fourier coefficients of an odd initial displacement function and, for the compression test model:

$$v^t(t,y) = \sum_{k=1}^{\infty} b_k \cdot \cos\left(\frac{(2k-1) \cdot \pi}{2H} y\right) \cdot e^{-\frac{(2k-1)^2 \cdot \pi^2}{4H^2} \cdot c_v \cdot t} \qquad (22)$$

where b_k (k=1...∞) are the Fourier coefficients of an even initial displacement function. The function u for the relaxation test model:

$$u(t,y) = \sum_{k=1}^{\infty} \alpha_k \left\{\left[\cos\left(\frac{k \cdot \pi}{H} y\right)\right] - 1\right\} e^{-\frac{k^2 \cdot \pi^2}{H^2} \cdot c_v \cdot t} \qquad (23)$$

and, for the compression test model:

$$u(t,y) = \sum_{k=1}^{\infty} \beta_k \sin\left[\frac{(2k-1)\pi}{2H} y\right] e^{-\frac{(2k-1)^2 \cdot \pi^2}{4H^2} \cdot c_v \cdot t} \qquad (24)$$

where

$$\alpha_k = a_k \frac{k\pi}{H}; \beta_k = -b_k \frac{(2k-1)\pi}{2H}; \ \alpha_0 = -\sum_{1}^{\infty} \alpha_k \qquad (25)$$

The following two dimensionless arguments R and T can be derived for the compression and for the relaxation test model, respectively:

$$Y = \frac{y}{H} \text{ or } Y = \frac{y}{2H} \qquad (26a,b)$$

$$T = \frac{ct}{H^2} \text{ or } T = \frac{ct}{4H^2} \qquad (27a,b)$$

The time factors is different for the two tests and, slower dissipation can be expected for the OCT than for the ORT at identical initial condition.

3.3 Qualitative behaviour of the consolidation models

To characterize the stress state within the sample, the solution of the Partial Differential Equations under the given boundary conditions was expressed in terms of the pore water pressure u solution. It was assumed that the pore water pressure solution u is monotonic.

3.3.1 Stress state in the sample

For the ORT model, the total stress σ is equal to the sum of a steady-state and a transient component. The final total stress σ_∞ depends on the displacement load v_0 and H sample height:

$$\sigma_\infty = \frac{E_{oed} v_0}{H} \qquad (28)$$

The latter is equal to the mean pore water pressure u_{mean}:

$$\sigma^t(t) = u_{mean}(t) = \frac{1}{H} \int_0^H u(t,y) \, dy \qquad (29)$$

Being the volume constant, the mean effective stress σ'_{mean} is constant being equal to the final total stress σ_∞. Therefore, the transient part of the effective stress is as follows:

$$\sigma'^t(t,y) = u_{mean}(t) - u(t,y) \qquad (30)$$

For a single-drained sample, it follows that for any value of the time variable (i) the effective stress σ' decreases with y having its minimum at the bottom (y=H), (ii) rebound (decrease in σ') takes place at the sample top (y=0), and compression (increase in σ') takes place at the sample bottom (y=H):

$$\varepsilon^t(t,y) = -\frac{1}{E_{oed}}\left[u_{mean}(t) - u(t,y)\right] \quad (31)$$

For the OCT model, the total normal stress is constant with time. The effective normal stress increases with time, since its transient part is equal to the pore water pressure and the consolidation model predicts basically the compression.

$$\varepsilon^t(t,y) = \frac{1}{E_{oed}} u(t,y) \quad (32)$$

The settlement can be expressed in terms of mean strain which depends on the mean pore water pressure as follows:

$$v(t,0) = \varepsilon_{mean}(t)H = \left[-\frac{u_{mean}(t)}{E_{oed}} + \frac{\sigma_0}{E_{oed}}\right]H \quad (33)$$

4 EVALUATION

The multistage oedometric relaxation test results were analysed using four different model-versions of the joint model H.

Model HC describes consolidation only, the model HCR includes relaxation, while HCRT and H describe the partial unloading effect on relaxation as well, with increasing parameter numbers (Table 2). These models use shape function H for the identification of the initial condition.

The four models for multistage oedometric compression tests were variants of the joint model BC consisting of a consolidation part-model and a creep part-model with the identification of the initial condition and the immediate compression (Table 3).

They are of complexity as follows: AC>A, BC>B, B>A. The simplest model A is the Terzaghi consolidation model, including the identification of the immediate settlement v_1.

Model B is the same as A, except that the initial condition is identified (using E type functions). Models AC and BC contain a creep part-model also and are similar to Bjerrum's model, as can be seen in Table 3.

The inverse problem solution method is presented eg., in (Imre et al, 2013).

Table 2. Relaxation test models, # of parameters (with partial unloading)*

	H	HCRT	HCR	HC
Consolidation Part	yes	yes	yes	yes
Relaxation Part	yes*	yes*	yes	no
Parameter #	*9*	*8*	*7*	*6*

Table 3. Compression test model, # of parameters

	BC	B	AC	A
Consolidation Part	yes	yes	yes	Yes
Creep Part	yes	no	yes	No
Parameter #	6	5	5	4

Table 41. Relaxation test model-variants, initial condition H, the mean fitting error [%] for 8 samples

Model-version	H	HCRT	HCR	HC
Mean fitting error {%]	1.24	1	1.29	2.29

Table 5. Compression test model-versions, the mean fitting error [%] for 8 samples

Model-version	BC	B	AC	A
Mean fitting error {%]	0.92	2.58	0.92	2.49

Table 6. MRT, parameters identified with model HCRT, mean coefficient of variation $\sigma(p^i)/p^i$ for 8 samples (last, long stage)

	c_v [m2/s]	σ_∞ [kPa]	s [-]	t_3 [s]
1	3E-08	395	0.04	1200
2	3E-08	606	0.06	120
$\sigma(p^i)/p^i$[%]	6.71	0.29	0.16	10.59

Table 7. MCT, parameters identified with model BC, mean coefficient of variation $\sigma(p^i)/p^i$ for 8 samples (last, long stage)

	c_v [m2/s]	v^i [cm]	$v^c=v_\infty$ [cm]	C_α [-]
1	2E-07	0,0041	0,0055	0,0039
2	4E-08	0,0619	0,0141	0,0067
$\sigma(p^i)/p^i$ [%]	16.55	4.41	2.04	3.29

5 RESULTS

A parallel model validation study was made for the MRT and the MCT. Several versions with different complexity levels of the suggested joint model were fitted to long test data to select the best model. The results are summarized as follows.

Tables 4, 5 show the main fitting results. The fitting error is reduced to about half if the relaxation/creep is taken into account. The parameter error for the MRT was smaller than for the MCT (Tables 6 to 7), considering the MRT long stages and the data measured during ‘closest’ stage of the MCT.

The identified initial condition (Fig. 1) was generally close to Terzaghi’s one for the compression test and was dependent on soil on soil plasticity for the oedometric relaxation test.

The coefficient of consolidation c_v identified for the MRT and MCT were comparable, the difference is probably within the uncertainty bound, the $c_v$was slightly larger for the joint models than for the consolidation models.

The identified coefficient of relaxation *s* and the coefficient of creep Cα were comparable with the literature data being slightly smaller.

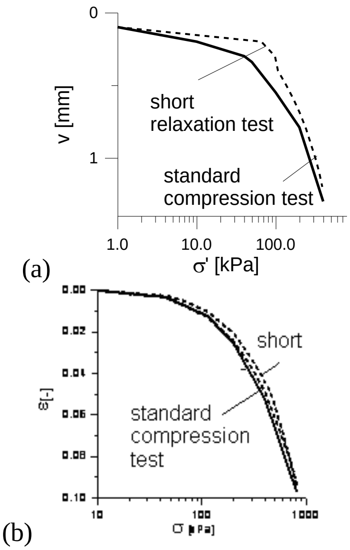

Figure2. Compression curves. (a) MRT. (b) MCT. In the short version the stages are 10 - 30 minute long, except the last stage that is longer than the t_{99} dissipation time, the long term compression curve point is determined by the inverse problem solution.

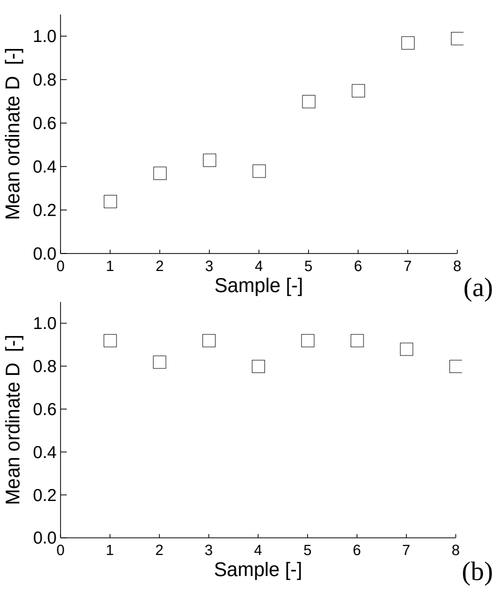

Figure 1. Parameter D identified from (a) the MRT test 1 to 8 (Note: in test 4 different load rate was applied.) (b) from the MCT test 1 to 8

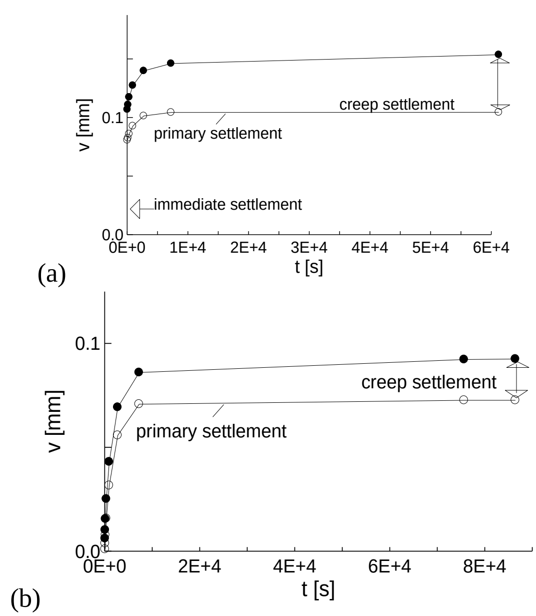

Figure 3. Compression test evaluation, data are related to soil 2 and 4, simulated and measured settlement, model AC. (a) sample 2. (b) sample 4.

According to the results (Figure 2), the short stage compression curves, were situated “above” the long stage compression curves, according to the expectations. Considering the simulated model-response of the MCT, the creep and consolidation settlements were about the same sizes (Figure 3).

6 DISCUSSION, CONCLUSION

Although several models have been suggested for the one dimensional oedometric compression test, they were seldom assessed experimentally (Leroueil et al, 1985). The conventional compression test is evaluated with the theory of Terzaghi in practice.

The evaluation method differs from evaluation methods reported in the literature in the following ways: (i) A joint model is validated considering creep and initial settlement; (ii) the inverse problem solution is automatic, mathematically precise, and includes reliability testing and regularization methods; (iii) the initial condition is identified.

The joint models are similar to the Bjerrum’s model where a linear uncoupled consolidation model and a creep model are superimposed (Bjerrum,.1967) and to the model - suggested by Lacerda and Houston 1973 for triaxial relaxation tests, resp.

The validation shows that (i) the fitting error decreases to half for those model-variants where the simultaneous creep/relaxation and the immediate compression are taken into account, (ii) the creep may qualitatively influence the model response and it is the most important at the initial part of the stress dissipation, and (iii) the short multistage test data can be successfully evaluated and, the testing time can be reduced with the doubled stage number.

The output of the evaluation method – instead of a single c value – may entail every parameter of the applied model (with reliability information. As a result, the settlement under a new building/dam can be estimated in terms of immediate, creep and consolidation components.

7 REFERENCES

Bjerrum, L. 1967. Engineering geology of normally consolidated marine clays as related to settlements. Seventh Rankine Lecture. Geotechnique, 162: 83-118.

Imre, E. 1995. Model discrimination for the oedometric relaxation test. Proc. of 8-th Baltic Geot. Conference, 55-60.

Imre, E. 1995. Model discrimination for conventional step-loaded oedometric test. Int. Symp. on Compression and Cons. of Clayey Soils, IS-Hiroshima'95, 525-530.

Imre, E. 1997 Consolidation models for incremental oedometric tests. Acta Tech. Acad. Sci. Hung. 369-398.

Imre, E. 1998. Evaluation of quick multistage oedometric relaxation tests. Proc. of the XIth Danube-European Conference on SMGE, Porec. 695-702.

Imre E, Schanz T, Hegedűs Cs 2013 Some thoughts in non-linear inverse problem solution. EURO:TUN Bochum.

Imre E, Schanz T, Hortobágyi Zs, Singh V P, Fityus S 2015. Oedometer relaxation test XVI ECSMGE Edinburgh, UK. 3351-3357. ISBN:978-0-7277-6067-8

Lacerda, W. A.; Houston W. N. 1973. Stress Relaxation in Soils. *Proc. of the 8th Inter. Conf. on Soil Mech. and Found. Eng. Moscow*, Vol.l. No.l. pp. 221-227

Leroueil, S.; Kabbaj, M.; Tavenas, F.; Bouchard, R. 1985. Stress-strain rate relations for the compressibility of sensitive natural clays. Geotechnique, Vol.35., No.2. pp. 159-175.

Terzaghi, K. 1923. Die Berechnung der Durchlassigkeitsziffer des Tones aus dem Verlauf hydrodyn. Spannung-sercsheinungen, Sitzber. Ak. Wiss. Wien, Abt.IIa, Vol. 123.