

# A note on seismic induced liquefaction

## Une note sur la liquéfaction sismique

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**ABSTRACT:** Any particle size distribution can be characterized by two parameter pairs (derived from the statistical entropy formula of discrete distributions) more effectively than simple diameter values (i.e.,  $d_{50}$ ). The first entropy parameter is a continuous internal stability measure. The second one allows the definition of a unique, mean grading curve. In this paper the seismically-induced liquefaction is examined on the basis of the case study of Kobe where gravel and silty sand contained gradings were liquefied. The entropy parameters and the internal stability criterion of the grading entropy theory indicated that the liquefied soils were internally unstable, while other criteria did not indicate possible liquefaction.

**RÉSUMÉ:** Toute distribution granulométrique peut être caractérisée par deux paires de paramètres (dérivées de la formule d'entropie statistique des distributions discrètes) plus efficacement que les valeurs de diamètre simple (c.-à-d.  $d_{50}$ ). Le premier paramètre d'entropie est une mesure de stabilité interne continue. Le second permet la définition d'une courbe de gradation moyenne. Dans cet article, la liquéfaction induite par sismique est examinée sur la base de l'étude de cas du séisme Kobe où le gravier et le sable limoneux contenaient des gradations étaient liquéfiés. Les paramètres d'entropie et le critère de stabilité interne ont indiqué que les sols liquéfiés étaient instables à l'intérieur, alors que d'autres critères n'indiquaient pas.

**Keywords:** Seismic liquefaction, static liquefaction, earthquake, grading entropy

## 1 INTRODUCTION

The potential for sands to liquefy can be evaluated according to the two classical liquefaction criteria of Tsuchida [1970] for soils with small and large uniformity coefficient, illustrated in Fig. 1. These are used in the

Technical Standards for Port and Harbour Facilities, published by the Japan Port and Harbour Association and in the design code for bridges in the USA. A similar criterion was by Smolczyk 2003, Numata and Mori 2004.

The Kobe earthquake occurred in 1995 and delivered a very high shock in the coast area.

The soils composed of granite gravel-containing silty sands developed extensive liquefaction in the reclaimed islands. These soils containing about 50% of gravels were not liquefying soils by the existing criteria (Fig. 1).

The liquefaction is a phenomenon observed in sands and silty sands being related to the positive pore water pressure development in the soil due to rapid stress oscillations associated with seismic events, which overcome inter-granular friction and reduce the soil's shear strength. The key elements are the small compressibility of grains material (granit or silica), the small permeability, the small density and – according to the assumption of this paper – the unstable internal structure.

This paper shows that the internal stability criterion of the grading entropy theory indicates that the soil is more prone to liquefaction liquefied if it is less stable internally even when the existing limit line criteria may fail.

As a physical explanation it can be said that the internal stability criterion may coincide the structural limit, where the sand internal structure changes from 'coarse in fine' to 'fine in coarse'. This limit can be linked with the maximum value of the minimum dry density  $s_{min}$  and the boundary of the small permeability zone.

## 2 GRADING ENTROPY

### 2.1 Definitions

#### 2.1.1 Fractions

An abstract fraction system is defined. The diameter range for fraction  $j$  ( $j=1, 2, \dots$ , Table 1) is:

$$2^j d_0 \geq d > 2^{j-1} d_0, \quad (1)$$

where  $d_0$  is the smallest diameter which may be equal to the height of the  $\text{SiO}_4$  tetrahedron. The 2 base log of the diameter limits are integers, called abstract diameters.

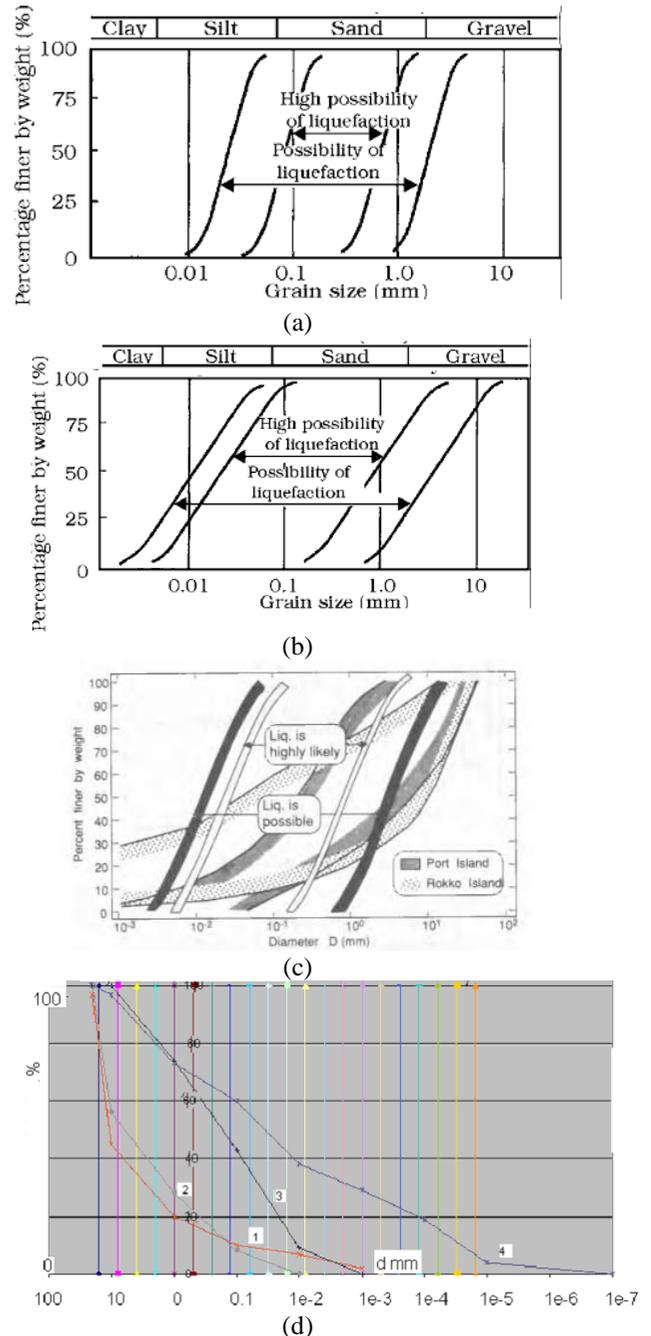


Figure 1. The liquefaction criterion of Tsuchida for (a) small uniformity coefficient (b) large uniformity coefficient. (c) Kobe soils (after Ishihara, 1999). (d) The Kob-gradings analysed in Table 3.

Table 1. Fractions

<i>j</i>	1	23	24
Limits	$d_o$ to $2 d_o$	$2^{22} d_o$ to $2^{23} d_o$	$2^{23} d_o$ to $2^{24} d_o$
$S_{oj}$ [-]	1	23	24

2.1.2 Grading entropy parameters

The grading entropy  $S$  is a statistical entropy, modified for the unequal cells (fractions are doubled). It can be separated into two parts (Lőrincz, 1986):

$$S = S_0 + \Delta S \tag{2}$$

where  $S_0$  is base entropy and  $\Delta S$  is entropy increment.  $S_0$  is a log “mean” of the diameter:

$$S_0 = \sum x_i S_{oi} = \sum x_i i \tag{3}$$

where  $S_{oi}$  is the  $i$ -th fraction entropy (Table 1). The entropy increment:

$$\Delta S = -\frac{1}{\ln 2} \sum_{x_i \neq 0} x_i \ln x_i \tag{4}$$

The relative base entropy  $A$  and normalized entropy increment  $B$ :

$$A = \frac{S_0 - S_{o\min}}{S_{o\max} - S_{o\min}} = \frac{\sum_{i=1}^N x_i (i-1)}{N-1} \tag{5}$$

$$B = \frac{\Delta S}{\ln N} \tag{6}$$

where  $S_{o\max}$  and  $S_{o\min}$  are the entropies of the largest and the smallest fractions, respectively.

2.2 Grading entropy map

Any particle size distribution can be represented by a point in the functions of the two entropy parameters which are maps between space of the gradings and the entropy diagrams.

2.2.1 Space of grading curves

The relative frequencies of the fractions  $x_i$  ( $i = 1, 2, ..N$ ) fulfil the following equation for every grading curve:

$$\sum_{i=1}^N x_i = 1, \quad x_i \geq 0, \quad N \geq 1. \tag{7}$$

where  $N$  is the number of the fractions between the finest and coarsest non-zero ones. The relative frequencies  $x_i$  can be identified with the barycentre coordinates of a point of an  $N-1$  dimensional simplex, representing the space of grading curves with  $N$  fractions.

The grading entropy parameter  $A$  is a linear function, the  $A = \text{constant}$  condition defines parallel hyper-plane sections of the  $N-1$  dimensional simplex, disjunct subspaces (Figs. 2-3). The grading entropy parameter  $B$  is a strictly concave function with a unique maximum for each  $A = \text{constant}$  value, which is a mean, “optimal” point or grading curve.

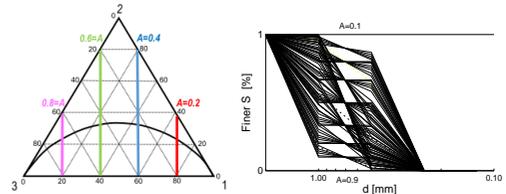


Figure 2. Representation of the 3-fraction soils ( $N=3$ ), with fixed  $A$  (left) in the triangle diagram (with the optimal line) and (right) as grading curves,  $A=0.1, 0.2, 1/3, 1/2$  and conjugate  $1-A$  values.

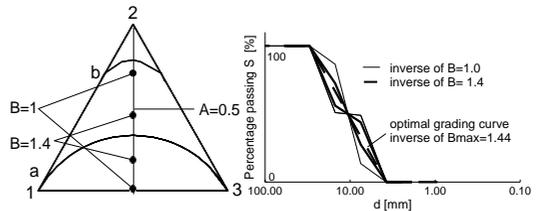


Figure 3. Representation of the 3-fraction soils ( $N=3$ ), with fixed  $A$  and  $B$  ( $A=0.66 B=1.2$  or  $A=0.5 B=1.2$  or  $A=0.3 B=1.2$ ) (left) in the triangle diagram as  $N-3=0$  dimensional circles, a pair of points) and (right) as pairs of grading curves.

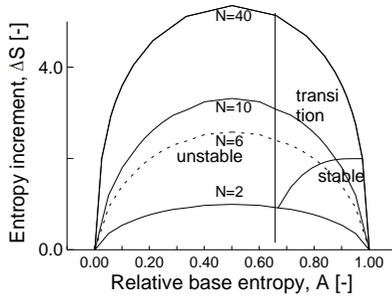


Figure 4. Partly normalised diagram with the internal grain structure stability criterion.

Table 2 Probability of  $A > 2/3$  in terms of  $N$

N [-]	2	3	10	50
P( $A > 2/3$ )	3E-01	1E-01	4E-02	2E-05

### 2.2.2 Entropy diagrams

Any grading curve can be represented as a single point in terms of the entropy coordinates (Fig. 4). Four maps can be defined between the  $N-1$  dimensional, open simplex (fixed  $N$ ) and the two dimensional real Euclidean space of the entropy coordinates, the non-normalized  $\Delta \rightarrow [S_0, \Delta S]$ , normalized  $\Delta \rightarrow [A, B]$ , partly normalized  $\Delta \rightarrow [A, \Delta S]$  or  $[S_0, B]$ .

The map for fixed  $N$  is continuous on the open simplex and can continuously be extended to the closed simplex. The entropy diagrams (Fig 4) are compact, like the simplex having a maximum line (which is about independent of  $N$  for a normalized diagram) and an approximate minimum line (the image of edge  $1-N$ ) which is dependent of  $N$  for a normalized diagram.

### 2.2.3 Optimal gradings

The inverse image of a maximum  $B$  value for a fixed  $A$  is a single point called optimal point or optimal grading curve, defined as follows:

$$x_1 = \frac{1}{\sum_{j=1}^N a^{j-1}} = \frac{1-a}{1-a^N}, \quad x_j = x_1 a^{j-1} \quad (8)$$

Table 3 Entropy parameters for Kobe soils (Fig.1(d))

N	21	14	16	12
$\Delta S$	4,22	3,68	2,49	3,15
$S_0$	12,75	15,95	15,07	18,93
A	0,54	0,53	0,47	0,72
B	1,39	1,39	0,90	1,27

where parameter  $a$  is the root of the equation:

$$y = \sum_{j=1}^N a^{j-1} [j-1 - A(N-1)] = 0 \quad (9)$$

The optimal grading curve at a given  $A$  are mean gradings for this  $A$  value having finite fractal grain size distribution, the fractal dimension (Imre and Talata, 2017):

$$n = 3 - \frac{\log}{\log} \quad (10)$$

The fractal dimension  $n$  varies between 3 and  $-\infty$  on the  $A > 0.5$  side of the diagram as  $a$  varies between 1 and  $\infty$ , moreover  $n$  varies between 3 and  $-\infty$  on the  $A < 0.5$  side of the diagram, as  $a$  varies between 1 and 0. The  $n$  depends on  $N$  except at the symmetry point ( $A = 0.5, B = 1/\ln 2, n = 3, a = 1$ ).

### 2.3 Internal stability

The internal stability criterion of the grading entropy was elaborated on the basis of vertical water flow tests in the partly normalized entropy diagram as follows (Figure 4).

In zone  $A < 2/3$  no structure of the large grains is present, the coarse particles “float” in the matrix of the fines and become destabilized when the fines are removed by piping. In zone  $A > 2/3$ , the structure is basically stable, for large  $N$  there is a transitional zone, where the stable structure is gradually set up.

The geometrical probability computable as ratio of the volume of the simplex where  $A > 2/3$  and the volume of the whole simplex tends to be zero with  $N$  (Table 2, Imre et al, 2018).

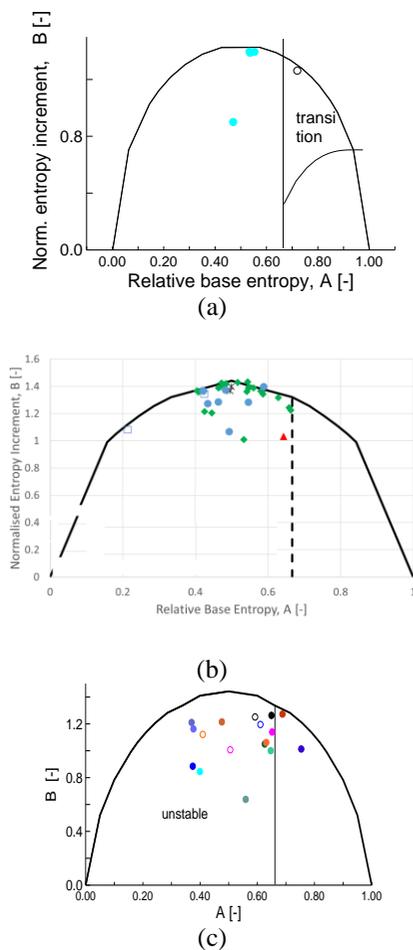


Figure 5. (a) The Kobe soils in the normalized entropy diagram. (b) 62 different gradings that have been reported Barreto et al, 2018 (c). Piping soils at Hungarian river sites.

### 3 KOBE EARTHQUAKE ANALYSIS

The fills beneath several island in the port area of Kobe were constructed using residual soil formed by weathering of granitic rocks. The soil dubbed Masado was obtained from borrow sites in the Rokko Mountain and brought to the site. This soil is collapsible in its natural state and its grain size distribution varies widely depending upon the degree of weathering.

The characteristic feature of the grain composition is that the material is generally well-graded and contains a significant fraction of gravel. Superimposed in Figure 1(c) are the ranges of gradation for soils which are likely to develop liquefaction if the soils are deposited in-situ under sufficiently loose conditions.

The ranges shown Figures 1(a) and (b) were established by examining the gradation curves of 2051 soils which showed liquefaction in past earthquakes. Comparing with these boundary curves, Kobe area were known to have grain composition outside the boundaries and hence considered not to be susceptible to liquefaction.

Based on the results of the entropy calculations (see Figure 5(a), Table 3), the position of the liquefied soils, reflect instability.

## 4 DISCUSSION

### 4.1 Liquefaction

#### 4.1.1 Barreto et al (2018) database

A database consisting of 62 different gradings that have been reported in the literature as liquefied are considered in Figure 5(b). All soils, including the gradation limits proposed by Tsushida are plotted, on area of the entropy diagram where the soils are unstable. Some cyclic triaxial experiments results and experimental data obtained from centrifuge tests were classified by the criteria of Tsushida as non-liquefiable soils.

#### 4.1.2 Piping database in Hungary

A combination of static and dynamic liquefaction may lead to piping, according to the result of a research related to the river dyke system in Hungary (Imre et al 2015 to 2018). The grain size distribution curve of the washed out soils along the rivers have a potential to liquefy by both the two criteria of Tsuchida (1970) and are internally unstable as shown in Figure 5(c).

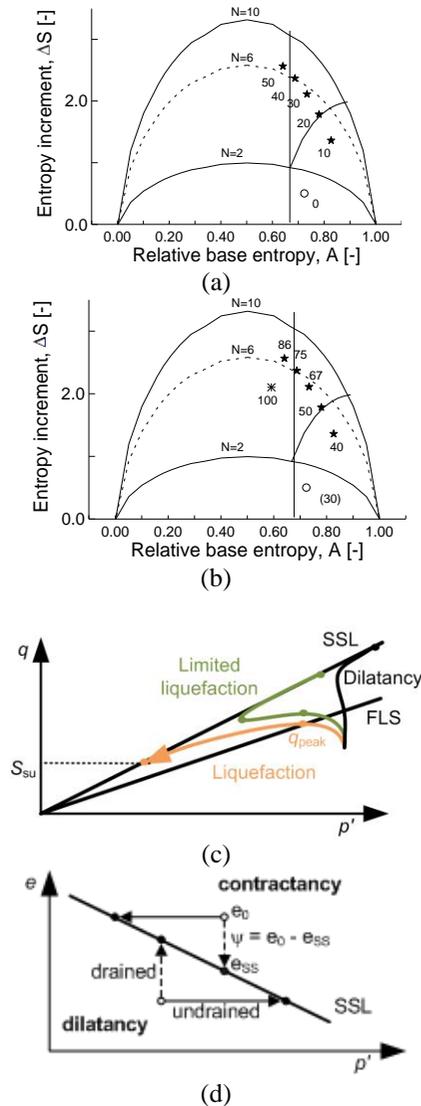


Figure 6. (a)-(b) Gratings with various fine content percentages, the related probability of static liquefaction. (Bracket: limited liquefaction.). (Rahemi 2017). (c)-(d) Schematic of possible stress paths in static undrained triaxial compression tests.

#### 4.1.3 Static liquefaction

The static liquefaction (Kramer and Seed, 1988) is studied in the function of fine content by Rahemi (2017). A series of 60 conventional triaxial compression tests are conducted on Hostun sand – silt mixtures to investigate the effects of fines on static liquefaction of sand.

In the tests, practically all possible initial relative densities are represented. The very first results indicate (Figure 6, Table 4) that – as the stable internal structure builds up, at fixed confining stress – the mixtures show decreasing liquefaction. The critical relative density and critical parameter  $\Psi$  decreases, the critical void ratio increases with decreasing fine content.

### 4.2 Internal structure

#### 4.2.1 Density in terms of $A$

The influence of fines on sand, from microstructure point of view, was first presented by Mitchell (1976). According to the results of Goudarzy (2015) shown in Figure 7, the maximum value of the minimum dry density of the sand-fine mixtures  $e_{max}$  is at around  $A=2/3$ , where the microstructure changed from ‘coarse in fine’ to ‘fine in coarse’ (Imre et al, 2018). This agrees the results of Lőrincz in a more general context ( ).eter value at around  $A=2/3$ .

#### 4.2.2 DEM in terms of $A$

Preliminary 3D DEM simulations of spherical particles with 2-fraction soils ( $N=2$ ) with various  $A$  values under drained triaxial conditions were tested. The results for this showed, as  $A$  increases, the specimens become inherently more stable (Imre et al, 2018).

#### 4.2.3 Permeability in terms of $A$

A correlation between the grading entropy coordinates and the coefficient of permeability – reflecting increasing void sizes with increasing  $A$  – is presented by Feng et al, 2018. According to Figure 8, the permeability and the large diameter pores are decreasing as  $A$  decreases.

Table 4. Critical relative density/ void ratio estimates at fixed confining stress with varying fine content %

	0	10	20	30	40	100
$D_r$ %	26	37	40	70	90	95
$e_{ss}$	0.93	0.80	0.70	0.64	0.48	0.53
$\Psi$	0.04	0.06	0.06	0.13	0.32	0.45

### 4.3 Pore water pressure due to penetration

The CPT dissipation test results show that in silty sands the pore water pressure is (real) negative with type V dissipation curves (Imre et al 2018a). In clays very large OC state is needed for a similar reply. It follows from the identification results of compression tests that the undrained deformability of the grains is different for sand and clay (Imre et al, 2013).

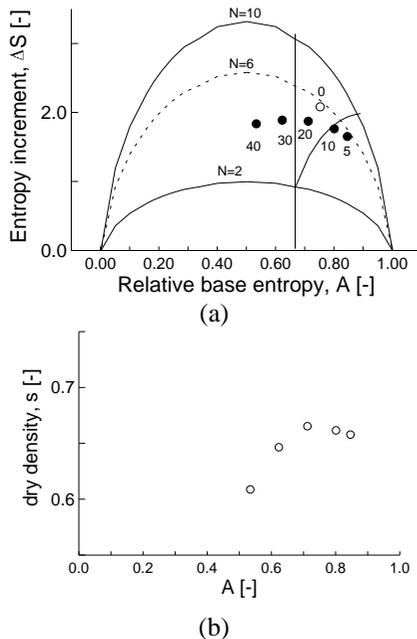


Figure 7. (a) Sand-fine mixtures with fine content,  $N=10$ . (b) The  $e_{max}$  results Goudarzy, 2015.

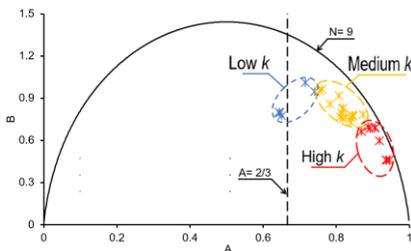


Figure 8 Permeability variation in normalised digram for grading curve series consisting of the same fractions, after Feng et al, 2018.

## 5 CONCLUSION

### 5.1 The entropy parameter and diagram

(1)The grading entropy parameters are various statistical means. The base entropy  $S_0$  is a kind of dimensionless mean log diameter. Its normalized version, the relative base entropy  $A$  is a normalized mean log diameter, varying between 0 and 1, the extremes are related to the minimum and maximum log diameters.

(2)The entropy increment  $\Delta S$  and its normalized version  $B$  are log weighted generalized geometrical means of the relative frequencies  $x_j$  ( $j = 1, \dots, N$ ), having maximum values of  $\ln N / \ln 2$  and  $1 / \ln 2$ , respectively. They reflect the actual effective number of fractions within the mixture. For those grading curves, in which all  $N$  fractions are well represented, the entropy increment is typically close to the maximum.

(2)Representing the space of the grading curves with  $N$  fractions by an  $N - 1$  dimensional, closed simplex, the  $A = \text{constant}$  condition means  $N - 2$  dimensional parallel hyper-planes in the Euclidean space generated by the simplex. The  $B = \text{constant}$  condition in addition is related to an  $N - 3$  dimensional, concentric topological circle around the optimal point on the  $A = \text{constant}$  hyper-plane section. The optimal point is defined by the maximum of  $B$  for the given  $A$ .

### 5.2 The internal stability

(1) The relative base entropy  $A$  has a potential to be a criterion number not only for internal stability, but also for several structural features based on the simple physical fact that if enough large grains ( $A > 2/3$ ) are present in a mixture then these will form a stable skeleton.

(2) The limit  $A=2/3$  is related to microstructural change, the internal structure changes from 'coarse in fine' to 'fine in coarse' gradually. The minimum dry density  $e_{max}$  has approximate maximum at  $A=2/3$  (Goudarzy, 2015, Imre et al 2018), and permeability start to increase with  $A$  from about here (Feng et al, 2018).

(3) Since the probability of an arbitrary mixture to be internally stable tends to be zero with  $N$ , if artificial soil are used for a construction then internally stable composition is suggested to be designed and mixed on the site. The segregation of an internally stable mixture is negligible (Lőrincz et al, 2015).

### 5.3 The liquefaction factors

(1) All liquefied soils were internally unstable by the entropy criterion, while the grain size based criteria did not indicate problematic features in the Kobe case.

(2) It can be noted that the small grain compressibility of sand-silt soils may support high excess pore water pressures and small permeability may sustain excess pore water pressure (Imre et al, 2018).

(3) In triaxial tests, the mixtures in the transition zone for a given confining stress and uniformly distributed initial density with decreasing  $A$  showed increasing static liquefaction (due to decreasing critical relative density and increasing critical void ratio). Further research is suggested on this.

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