Proceedings of the XVII ECSMGE-2019
Geotechnical Engineering foundation of the future
ISBN 978-9935-9436-1-3

doi: 10.32075/17ECSMGE-2019-0961

Shear Localisation Analysis of Drained Sand with Nonlocal Regularisation Method

Analyse de localisation par cisaillement de sable drainé avec une méthode de régularisation non locale

H.E. Mallikarachchi,
University of Cambridge,Cambridge,United Kingdom

K. Soga
University of California, Berkeley, United States

ABSTRACT: The finite element method suffers from mesh dependency and numerical errors during modelling of strain localisation. Standard constitutive models assume that the mechanical behaviour at a point is dependent on the current deformation at that individual point only. In the event of a shear localisation, an individual material point can experience a larger strain than the neighbourhood resulting in a higher strain gradient. In the strain softening materials, this leads to a greater reduction in strength in an individual point. Hence the governing partial differential equations loose ellipticity. Among the various types of regularisation used to circumvent this drawback, the nonlocal method is popular due to its simplicity and versatility.

The focus of this study is to apply the nonlocal regularisation technique to simulate the shear localisation observed in biaxial compression tests of dilative dense sand, under drained boundary conditions. A linear softening Drucker-Prager (DP) model is enriched with the nonlocal theory and its capability to evade the mesh sensitivity observed. Finally, the softening scaling is introduced to scale the load displacement response. The ultimate aim is to obtain a realistic force displacement response during the shear localisation of dense sand without extreme mesh refinement.

It is observed that the load displacement relationships are independent of the mesh size after the nonlocal regularisation. The numerical shear band thickness is insensitive to the mesh size and instead proportional to the adopted characteristic length. This results in reducing the softening rate thus minimizing numerical convergence issues. However, the physical thickness of the shear band in sand is equivalent to several particle diameters. Hence, the softening scaling is used to rectify the regularised force displacement curves to match with the realistic softening rate of sand.
Keywords: shear localisation; mesh dependence; nonlocal regularisation; drained sand

RÉSUMÉ: La méthode des éléments finis souffre de la dépendance au maillage et d'erreurs numériques lors de la modélisation de la localisation des contraintes. Les modèles constitutifs standard supposent que le comportement mécanique en un point dépend de la déformation actuelle à ce point seulement. Dans le cas d'une localisation de cisaillement, un point matériel individuel peut subir une contrainte plus importante que le voisinage, ce qui entraîne un gradient de contrainte plus élevé. Dans les matériaux d'assouplissement des contraintes, cela conduit à une réduction plus importante de la résistance d'un point individuel. Par conséquent, les équations aux dérivées partielles qui gèrent l'ellipticité sont lâches. Parmi les divers types de régularisation utilisés pour contourner cet inconvénient, la méthode non locale est populaire en raison de sa simplicité et de sa polyvalence.

L'objectif de cette étude est d'appliquer la technique de régularisation non locale pour simuler la localisation de cisaillement observée lors d'essais de compression biaxiale de sable dense et dilaté dans des conditions limites drainées. Un modèle de Drucker-Prager (DP) à adoucissement linéaire s'enrichit de la théorie non locale et de sa capacité à échapper à la sensibilité du maillage observée. Enfin, la graduation de ramollissement est introduite pour redimensionner la réponse au déplacement de charge. Le but ultime est d'obtenir une réponse de déplacement de force réaliste lors de la localisation par cisaillement de sable dense sans raffinement extrême des mailles.
On observe que les relations de déplacement de charge sont indépendantes de la taille du maillage après régularisation non locale. L'épaisseur numérique de la bande de cisaillement est insensible à la taille du maillage et proportionnelle à la longueur caractéristique adoptée. Cela permet de réduire le taux de ramollissement, minimisant ainsi les problèmes de convergence numérique. Cependant, l'épaisseur physique de la bande de cisaillement dans le sable est équivalente à plusieurs diamètres de particules. Par conséquent, l'échelle de ramollissement est utilisée pour rectifier les courbes de déplacement de force régularisées afin de correspondre au taux de ramollissement réaliste du sable.

1 INTRODUCTION

Finite element modelling of strain localisation frequently suffers from pathological mesh dependence and numerical errors from lack of convergence. Solutions are sensitive to the spatial discretisation such as alignment and size of mesh. In typical finite element analysis, displacements are calculated at nodes, thus their relative locations govern the shear band thickness and its direction.

The reason underlies the hypothesis of treating the material as a continuum of arbitrary small scale made out of a set of infinitesimal material volumes, of which constitutive behaviour is independent. However, in an ideal material such as soil, the microstructure varies over different orders of magnitude. In granular material, the thickness of intense shearing is experimentally proved to be the size of several particle diameters. It serves as a measure of the physical thickness or the resolution of the soil.

The conventional continuum approach is adequate if the characteristic wavelength of the deformation field remains above the resolution level of the material (Bažant & Jirásek, 2002). However, available limited computational resources forbid to refine the mesh to match the physical characteristic length of the shear band. Hence, the characteristic wavelength of the field (shear band thickness) is always below the resolution level of the model (element size). Therefore, the post-bifurcation response of a finite element continuum is inherently mesh dependent.

Standard constitutive models assume that the mechanical behaviour at a point is dependent on the current values and the previous history of deformation at that individual point only. In the event of a shear localisation, an individual material point can experience a larger strain than the neighbourhood resulting in a higher strain gradient. In strain softening materials this leads to a greater reduction in strength in that point only. Hence, governing partial differential equations change from elliptic to hyperbolic (for static problems) resulting mathematically in ill-posed boundary value problems.

Enrichment techniques are used to regularise the mesh dependence of the finite element method by introducing a characteristic length which is related to the physical shear band thickness. The objectivity is achieved by bridging the gap between soil microstructure and the continuum. Techniques used for the post-bifurcation analysis in past few decades fall into several categories. Depending on the method, the internal length scale is embedded either in the constitutive model itself (nonlocal, viscous plasticity) or equilibrium equations (Cosserat theories, gradient theories, micro-polar theories) (Bažant & Jirásek, 2002).

2 NONLOCAL THEORY

From potential regularisation techniques used for the post-bifurcation analysis, nonlocal methods are numerically efficient and easy to implement. This is because they can be applied to the constitutive level without altering equilibrium equations.

The nonlocal approach assumes that micro kinematics of a singular point influence surrounding points as well. The concept of the nonlocal continuum was proposed for elasticity by Eringen (1966), Eringen and Edelen (1972). The original philosophy was introducing the nonlocal character to many fields such as stress, mass, body forces and energy which was later termed as the fully nonlocal theory. However, they were too complicated to be implemented in finite element formulations. Later Eringen and Kim (1974) simplified the theory considering only the constitutive relationship as nonlocal while equilibrium and kinematic equations remain unaltered. The stress at a point ($\boldsymbol{\sigma}$), is considered as a function of mean strain ($\bar{\boldsymbol{\varepsilon}}$) averaging over a representative volume centred at that point (V).

The most recent adoption to the nonlocal softening plasticity is to treat only the scalar softening variable (κ) as nonlocal. This method exhibits to be computationally efficient and reduces the mesh locking as well (Bažant & Jirásek, 2002). This kind of approach is termed as the partially nonlocal theory. This was later adopted for soil plasticity to simulate the shear localisation. The softening parameter which drives the yield stress degradation was treated nonlocal as explined by Equations 1 and 2 (Brinkgreve, 1994),(Summersgill, Kontoe, & Potts, 2014) ,(Summersgill, Kontoe, & Potts, 2017) ,(Vermeer & Marcher, 2000).

$$\sigma = \sigma_0 + h\,(\bar{\kappa}\,) \qquad (1)$$

$$\bar{\kappa}(x) = \frac{1}{V}\int_V \; w(x,\xi)\,\kappa(\xi)\;d\xi \qquad (2)$$

x and ξ are global and local coordinates respectively and are shown in Figure 1. w is the weight function which depends on the characteristic length (l)

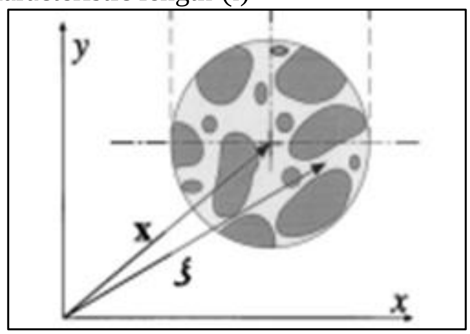

Figure 1. Representative volume used for nonlocal strain averaging

The most commonly used weight function in the nonlocal theory is the Gaussian distribution (Equation 3) which has a maximum at the centre. It gives the greatest contribution of the averaging variable at the centre. This will ultimately lead to a central concentration of the softening variable resulting in a drastic loss in material strength at a solitary point. Several attempts have been made to address this issue by altering the weight function or the averaging procedure itself.

Galavi and Schweiger (2010) introduced a weight function in Equation (4) which is termed as GandS method. In this, the contribution of the softening variable at the considered point is zero. This function has two peaks such that the greatest contribution is from either sides of the central point. Over nonlocal method introduced by Brinkgreve (1994) altered the nonlocal averaging formulation itself to circumvent the concentrated peak at the centre (Equation 5). This smears the averaging variable in the neighbourhood without changing the weight function, but it introduces the over nonlocal parameter (m) additionally.

$$w(x,\xi) = \frac{1}{\sqrt{\pi}\, l} exp\left[\frac{-(x-\xi)^2}{l^2}\right] \quad (3)$$

$$w(x,\xi) = \frac{(x-\xi)}{l} exp\left[\frac{-(x-\xi)^2}{l^2}\right] \quad (4)$$

$$\bar{\kappa}(x) = (1-m)\,\kappa + m\,\frac{1}{V}\int_V \; w(x,\xi)\,\kappa(\xi)\; d\xi \quad (5)$$

3 IMPLEMENTATION OF NONLOCAL DRUCKER-PRAGER MODEL

Formulations for simple extension of friction softening to traditional Drucker-Prager (DP) model is described herein. DP yield surface is defined as,

$$F = q - \alpha\, p \quad (6)$$

Where p and q are mean pressure and deviatoric stress respectively. Since the DP model creates a circular yield surface in the deviatoric plane, friction angle is independent of the lode angle. Thus, the numerical implementation is carried out as a two invariant model. A linear softening rule is defined such that friction coefficient (α) is reduced with the deviatoric plastic strain (γ^p).

$$\alpha = \alpha - h_\alpha\, d\gamma^p > \quad \alpha_{cs} \quad (7)$$

Where h_α is the softening modulus and α_{cs} is critical friction coefficient. A non-associative plastic flow rule is introduced as,

$$d\varepsilon^p = d\lambda\, \frac{\partial G}{\partial \sigma} \quad (8)$$

$$G = q - \beta\, p \quad (9)$$

Where the plastic potential (G) is related to dilation coefficient (β).

A partial nonlocal theory is adopted in this study, which makes only the softening parameter nonlocal. The local plastic deviatoric strain increment in Equation 7 is replaced with the weighted average of plastic deviatoric strain increment $(d\gamma^p_{avg})$.

$$\alpha = \alpha - h_\alpha\, d\gamma^p_{avg} \quad (10)$$

The spatial average is calculated from the discretised form of the integral in Equation 2.

$$d\gamma^p_{avg} = \frac{\sum_{k=1}^{n} v_k\, w_k\, d\gamma^p_k}{\sum_{k=1}^{n} v_k\, w_k} \quad (11)$$

Where n is the number of integration points, v_k and w_k are representative volume and weight function of the integration point k. The aforementioned weighted average is only taken within the radius of 3l from the integration point. This is because the magnitude of weight function (w_k) becomes insignificant beyond this distance.

4 MESH COMPARISON ANALYSIS

Numerical biaxial compression simulations with four mesh sizes are conducted to assess the validity of the nonlocal regularisation. The modified constitutive model is implemented in ABAQUS user-defined material model. The information of the gauss points in the neighbourhood is accessed through the common block option.

The biaxial specimen used in this study is 25cm wide and 50cm high. The plane strain condition assumed throughout. The bottom boundary conditions are such that the leftmost node is pinned and other nodes are roller supported. Top or side boundaries are not restrained. Weak material points are included at the bottom right corner. This facilitates a formation of single shear band diagonally across the specimen.

During the first step, the specimen is consolidated homogeneously with a confining pressure of 100 kPa. During the second step, a vertical displacement is applied to the top nodes to allow an axial compression.

The analysis comprises a comparison of four meshes with different element sizes. They are named as large (100 elements), medium (800 elements), small (1250 elements) and extra small (3200 elements) for mesh sizes 0.025m, 0.0125m, 0.01 m, 0.00625 m respectively. In all cases, plane strain quadratic isoparametric elements with reduced integrations (8 nodes and 4 integration points - CPE8R) are used. The characteristic length for the nonlocal models is selected equal to the largest mesh size $(0.025m)$. The over nonlocal parameter is selected as 2.

Dense sand with an initial void ratio of 0.55 is used for all the analysis. The elastic stiffness, E

is assumed to be varying with the mean pressure accroding to the relationship $E = Ap^n$, where A and n are specified as 800 and 0.5 respectively. The poisson ratio is 0.3. Peak and critical friction coefficients are 1.2 and 0.8 respectively. The dilation coefficient is assumed to be a constant 0.47. The softening modulus is 0.7.

5 RESULTS AND DISCUSSION

5.1 Force displacement curves

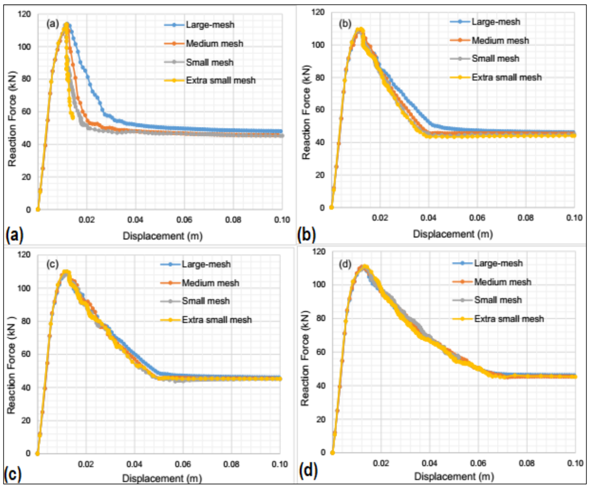

Figure 2. Force displacement relationships for (a) local DP model and nonlocal DP models with (b) Gaussian weight function (c) GandS weight function, (d) over nonlocal model with parameter 2

According to Figure 2(a), the force displacement responses of local DP model is mesh insensitive up to the peak and become mesh dependent afterwards. Both peak and residual strengths are mesh independent, but the softening rate becomes greater when the mesh size is reduced. The extra fine mesh fails to converge during the softening due to the sudden strength reduction of individual material points. This is ascribed to the loss of positive definiteness of the global stiffness, which is shown by almost vertical or snapback behaviour of the load curve.

Force-displacement responses of nonlocal DP models from three methods in Figure 2 (b), (c) and (d) show a considerable mesh independence compared to those of the local DP model. The peak and the residual are unchanged whereas softening rates (stiffness) are reduced. As the drastic reduction of strength at an individual material point is diminished, numerical convergence difficulties are minimized. This aids even the extra fine mesh to be fully converged.

When comparing three methods, the original nonlocal method with Gaussian distribution appears to be less productive in regularising the localised shear strain as shown in Figure 2(b). This is because the standard Gauss distribution allocates the highest weight in the centre allowing the strain concentration within the localised area to be still higher. This is circumvented by both GandS and over nonlocal methods. It is also noted that the softening stiffness varies with the method regardless of the same characteristic length. The softening rate is affiliated with the width of the shear band which is numerically related to the characteristic length, the weight function and the over nonlocal parameter. Both GandS and over nonlocal methods bestow larger shear band thickness and hence lower softening rates than the original nonlocal method with the Gaussian weight function.

The pitfall of the over nonlocal method is that the over nonlocal parameter has to be selected from trial and error based on the characteristic length and mesh size such that an acceptable shear band thickness results. Therefore, for further comparison of local vs nonlocal DP models, GandS method is selected since it simply changes the weight function facilitating a smoother distribution of strain without an additional parameter.

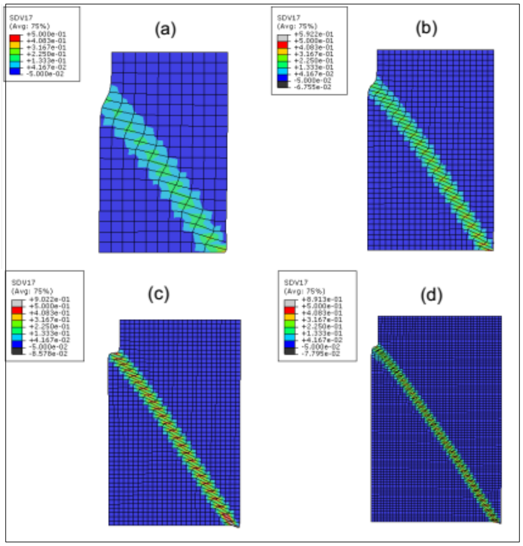

Figure 3. Contours of accumulated deviatoric plastic strain from local DP model for (a) large, (b) medium, (c) small and (d) extra small meshes

5.2 Shear band thickness

Contours of accumulated deviatoric plastic strain (during post-peak softening) of both local and nonlocal analysis are displayed in Figures 3 and 4 respectively. In the local analysis, both thickness and magnitude of shear strain concentration are sensitive to the mesh size. Although extra fine mesh does not converge beyond 0.015m, a considerable shear deformation has taken place by that time. On the contrary, the width of shear band thickness has become almost mesh independent after nonlocal regularisation with GandS weight function. However, the magnitude of maximum shear strain inside the shear zone still appears to be mesh dependent.

Regarding the orientation of the shear band, it is apparent that finer the mesh, higher the shear band angle for local analysis. After the regularisation, the shear band angle appears to be closely mesh independent except for the large mesh.

The thickness of the shear band can be evaluated based on either change in displacement gradient or strain fields. As an example, the cross-sectional profiles of displacement and strain in the large mesh are shown in Figures 5 and 6 respectively. The choosen cross-section is perpendicular to the shear band starting from coordinates (0,0.15). Three stages of deformation are included to get an insight into the evolution of the shear band. A clear distinction can be seen between pre-peak homogeneous deformation vs post-localised deformation.

Apparently, the shear zone enlarges with the deformation until it stabilises at the residual state. This is attributed to the expansion of dilating shear band material. The position and orientation of the shear band also vary slightly with the deformation.

Further, it is evident that the nonlocal regularisation smears the shear strain concentration over larger thickness reducing the peak. This is responsible for the lowering of softening stiffness.

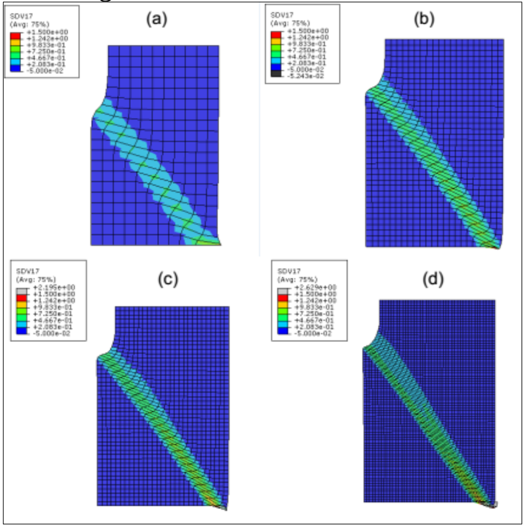

Figure 4. Contours of accumulated deviatoric plastic strain from nonlocal (GandS) DP model for (a) large, (b) medium, (c) small and (d) extra small meshes

The mechanism behind the nonlocal DP model is the regularisation of the friction coefficient, which acts as the softening parameter in the DP model.

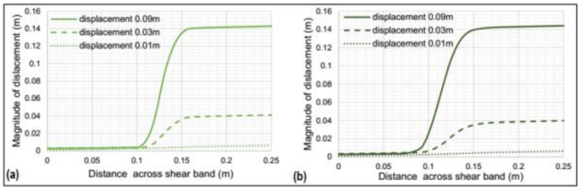

Figure 5. Cross-sectional profiles of displacement of large mesh: (a) local and (b) nonlocal method

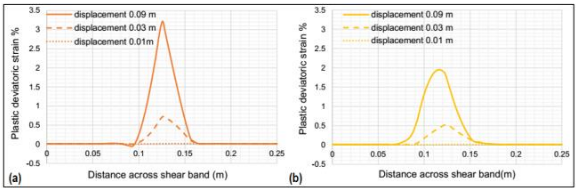

Figure 6. Cross-sectional profiles of plastic deviatoric strain of large mesh:(a) local and (b) nonlocal method

Cross-sectional plots of the friction coefficient in Figure 7 (right) depict that after the nonlocal regularisation, both magnitude and the width of the friction coefficient are mesh independent. It is important to note that in the local analysis, material points inside the shear band have already reached the residual state when the vertical displacement is 0.03m. On the contrary, in the nonlocal analysis, material points are still softening by the displacement of 0.03m. This portrays how the drastic reduction of strength is minimized by the nonlocal regularisation. Further, fluctuations of the friction coefficients in the border of the shear bands in Figure 7 (left) are diminished after regularisation.

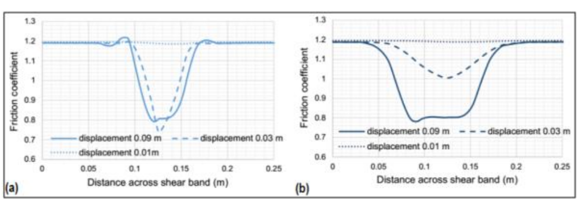

Figure 7. Cross-sectional profiles of friction coefficient of large mesh: (a) local and (b) nonlocal method

The correlation between the shear band thickness and the mesh size is plotted in Figure 8(a) for local analysis. It is obverved that the numerical shear band is almost twice the meh size. Figure 8(b) encapsulates results of several nonlocal analysis with different characteristic lengths. Apparently, the numerical shear band thickness obtained from the nonlocal analysis is also closely twice the characteristic length.

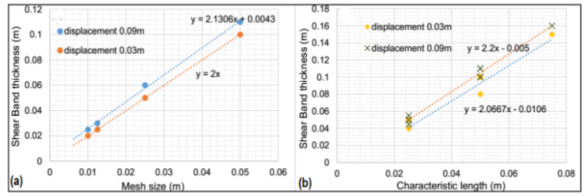

Figure 8. Correlation between shear band thickness and (a) mesh size in local analysis and (b) characteristic length in nonlocal analysis

5.3 Softening scaling

The nonlocal method grants the user the power of selecting the softening stiffness with an appropriate characteristic length such that the well-posedness of the boundary value problem is preserved. However, Figure 3 suggests that the the nonlocal method produces unrealistically large numerical shear band thickness (t_{num}) reducing the softening rate. Therefore, the resulting load-displacement curves do not represent the realistic soil behaviour. This is because the used characteristic length is much larger than the physical length of sand. If the actual length scale of sand is used here, the mesh size has to be lower than that for the nonlocal method to be effective. Nonetheless, it is practically impossible to refine the mesh to comply with the real shear band thickness.

To overcome this drawback, Brinkgreve (1994) proposed to scale the physical softening stiffness (h_{sand}) such that the resulting mechanical response complies with the real soil (Equation 12). Experimental studies suggest that the physical shear band thickness of sand (t_{sand}) is $10 - 20\ D_{50}$. Assuming that D_{50} is 1 mm, the physical shear band thickness is 10 mm. Thus the scaling factor of softening stiffness is calculated to be 5 for the characteristic length of 0.025m. Thus, the modified softening stiffness is 4000.

$$h_{num} = h_{sand} \frac{t_{num}}{t_{sand}} \qquad (12)$$

The nonlocal DP model after scaling is shown in Figure 9. It is evident that the scaling introduces a drastic softening rate such that the peak strength reduces to the residual almost instantly. There is no change in either peak or residual strengths. The finer the particle size (smaller actual shear band thickness), the steeper softening stiffness becomes. Hence, this method is only applicable only to soil with finite shear band thickness.

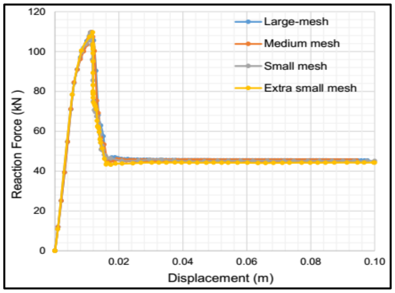

Figure 9. Force displacement relationship of nonlocal analysis after softening scaling

6 CONCLUSION

Nonlocal regularisation sufficiently alleviates the mesh obejctivity of post-localised deformation of drained dense sand. GandS and over nonlocal methods provide more reliable regularisation than the original Gaussian weight function. The nonlocal averaging smears the shear strain concentration widening the shear zone. The numerical shear band thickness is a function of characteristic length after the regularisation. The softening scaling can be utilised to scale the physical softening stiffness such that the regularised mechanical response matches with the real soil behaviour. Hence the characteristic length of the nonlocal method can be chosen arbitarily larger than the physical shear band thickness. Therefore, the post localised deformation can be captured with sufficient accuracy without extreme mesh refinement. However, this method is successful only for soils with a finite shear band thickness, under the assumption of weak discontinuity.

7 REFERENCES

Bažant, Z. P., & Jirásek, M. (2002). Nonlocal Integral Formulations of Plasticity and Damage: Survey of Progress. *Journal of Engineering Mechanics, 128*(11), 1119-1149.

Brinkgreve, R. (1994). Geomaterial models and numerical analysis of softening. *Technische Universiteit Delft*(April 1994), 169.

Eringen, A. C. (1966). A unified theory of thermo mechanical materials. *International Journal of Engineering Science, 4*, 179-202.

Eringen, A. C., & Edelen, D. G. (1972). Non local elasticity. *International Journal of Engineering Science*, *10*, 233-248.

Eringen, A. C., & Kim, B. S. (1974). Stress concentration at the tip of a crack. *Mech. Res. Commun.*, 233-237.

Galavi, V., & Schweiger, H. (2010). Nonlocal Multilaminate Model for Strain Softening Analysis. *International Journal of Geomechanics, 10 (1)*(February), 30-44.

Summersgill, F. (2014). *Numerical modelling of stiff clay cut slopes with nonlocal strain regularisation.*

Summersgill, F., Kontoe, S., & Potts, D. (2017). Critical Assessment of Nonlocal Strain-Softening Methods in Biaxial Compression. *International Journal of Geomechanics*, 1-14.

Vermeer, P., & Marcher, T. (2000). Macromodelling of softening in non cohesive soils. In P. Vermeer, T. Marcher, P. Vermeer, S. Diebels, W. Ehlers, H. Herrmann, S. Luding, & E. Ramm (Eds.), *Continous and discontinous modelling of cohesive frcitional material* (pp. 89-108). Stuttgart: Springer.