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Simulation of quasi-static collapse of cylindrical granular columns, insight from continuum and discrete frameworks

Simulation du collapse quasi-statique de colonnes granulaires cylindriques, approches des mécaniques des milieux continus et discrets

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ABSTRACT: Collapse of granular columns subjected to different flow conditions has recently been the topic of numerous experimental and numerical studies. The quasi-static collapse of cylindrical granular columns is an interesting case of granular flow relevant to the active deformation of retaining structures. In this study, this type of collapse is investigated using Smoothed Particle Hydrodynamics (SPH) method in the continuum framework and Discrete Element Method (DEM) in the discrete framework. Three-dimensional (3D) SPH and DEM models are developed to study the dependency of collapse pattern, final deposit profile (e.g., height, runout distance), and energy dissipation on the initial aspect ratio of cylindrical granular columns. Results demonstrate that by using appropriate constitutive models or contact behaviors, both SPH and DEM models are qualitatively and quantitatively capable of capturing different aspects of quasi-static granular column collapse. However, where the particle-level behaviors dominate (e.g., at flow front and sharp edges of free surface, tracing frictional loss at contact), DEM can be considered as the method of choice.

RÉSUMÉ: Le collapse des colonnes granulaires soumises à différentes conditions d'écoulement a récemment fait l'objet de nombreuses études expérimentales et numériques. Le collapse quasi-statique des colonnes granulaires cylindriques est un cas intéressant d'écoulement granulaire concernant la déformation des structures de soutènement. Dans cette étude, ce type de d'effondrement est étudié par l'hydrodynamique des particules lisses (SPH, Smoothed Particle Hydrodynamics) dans le cadre de la mécanique des milieux continus et par la méthode des éléments discrets (DEM, Discrete Element Method) dans le cadre de la mécanique des milieux discrets. Des modèles SPH et DEM tridimensionnels (3D) sont développés pour étudier la dépendance du type de motif d'effondrement, du profil de dépôt de particules final (par exemple, la hauteur et distance finale) et de la dissipation d'énergie sur le facteur de forme initial des colonnes granulaires cylindriques. Les résultats démontrent que les modèles SPH et DEM sont qualitativement et quantitativement capables de capturer différents aspects du collapse quasi-statique de la colonne granulaire si des lois de comportement et de contacte appropriées sont utilisées. Cependant, sur les fronts d'écoulement et surfaces libres où le comportement des particules est dominant, la méthode des éléments discrets est préconisée.

Keywords: Discrete Element Method; Energy Dissipation; Granular Column Collapse; Quasi-Static flow; Smoothed Particle Hydrodynamics

1 INTRODUCTION

Flow of granular materials is encountered in various natural phenomena (e.g., debris avalanches and landslides) and industrial processes (e.g., chutes, and conveyer belts). The complexity of their behavior, which varies between solid-like to fluid-like, makes their study specifically challenging. Two important experimental studies on dynamic collapse of granular columns (i.e. Lube et al. 2004 and Lajeunesse et al. 2004) were conducted to study the effect of different factors including aspect ratio of granular column (initial height/initial radius) and materials properties (e.g., friction angle and density) on overall dense granular flow behavior. Results of these experimental studies revealed that initial aspect ratio of the column plays significant role on the final deposit profile (final height and runout distance) and material properties have negligible effects. These experimental studies had been used as an integral part of several numerical studies in continuum framework (e.g. Chen and Qiu 2012; Zhang et al. 2014; Mast et al. 2015) and discrete framework (e.g. Huang et al. 2013; Kermani et al. 2015).

An important yet underappreciated type of granular flow is quasi-static deformation of granular materials, which is relevant to phenomena such as active deformation of retaining walls and bridge abutments. During quasi-static flow, particle-particle contact is present throughout the flow, and the effect of inertia is negligible. Experimental studies on this type of flow is limited to work of Meriaux (2006), during which slow deformation of a rectangular granular column was studied. Results of that experimental study had been used for calibration and verification of few numerical studies by (2D PFEM by Zhang et al. 2016; 2D and 3D Discrete Element Method by Owen et al. 2008 and 3D Smoothed Particle Hydrodynamics by Kermani and Qiu 2018a,b). The results demonstrated that similar to dynamic collapse, the initial aspect ratio of the columns is the most significant factor in the quasi-static collapse. However, final runout distance is constantly smaller in quasi-static collapse.

All aforementioned experimental and numerical studies of quasi-static collapse were conducted on rectangular columns. In this study, focus has been placed on numerical modeling of quasi-static axisymmetric collapse of cylindrical granular column from continuum and discrete frameworks, since due to difficulties in conducting this test in laboratory, no experimental study has been reported so far.
Discrete Element Method (DEM) from discrete framework and Smoothed Particle Hydrodynamics method (SPH) from continuum framework have been utilized to simulate the quasi-static axisymmetric collapse of granular columns. The 3D DEM and SPH models were initially developed by Kermani et al. (2015) and Kermani and Qiu (2018a,b) and validated using readily available experimental studies in literature (e.g., Lube et al. 2004 and Meriaux 2004). These models provide invaluable insights into the axisymmetric quasi-static flow of granular columns, which is difficult for laboratory experimentation from both continuum and discrete perspectives and highlight the ability of these different schemes in modeling granular material flow in terms of final deposit profile and energy dissipation. In addition, this study provides a comprehensive comparison between the capabilities of SPH and DEM simulation techniques from continuum and discrete frameworks in modeling granular materials behaviors.

2 NUMERICAL METHODS

2.1 *Smoothed Particle Hydrodynamics (SPH)*

SPH is a robust particle-based Lagrangian mesh-free simulation technique, initially proposed by Gingold and Monaghan (1977) and Lucy (1977) for astrophysical studies, in which the problem domain is represented by a series of particles each carrying various properties (e.g. mass(m), volume(V), density(ρ), pressure(P), stress(σ), velocity(v), etc.). These particles interact with neighboring particles within their influence domain through a smoothing/weighting function W(e.g., cubic spline kernel function Monaghan and Lattanzio (1985)), and follow continuity and momentum conservation equations as (Liu and Liu 2003):

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j\left(v_i^{\alpha} - v_j^{\alpha}\right)\frac{\partial W_{ij}}{\partial x_i^{\alpha}} \qquad (1)$$

$$\frac{Dv_i^{\alpha}}{Dt} = \sum_{j=1}^{N} m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x_i^{\alpha}} - \sum_{j=1}^{N} m_j \Pi_{ij}\frac{\partial W_{ij}}{\partial x_i^{\alpha}} + g_z \qquad (2)$$

where $\frac{D}{Dt}=$material derivative; $t=$time; $i=$index for target particle; $j=$index for neighboring particles; $N=$ total number of j particles; $g_z=$gravity; $\Pi_{ij}=$artificial viscosity term for improving SPH stability (Liu and Liu 2003); total stress tensor $\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + s^{\alpha\beta}$; $P = -(\sigma^{xx} + \sigma^{yy} + \sigma^{zz})/3$ is isotropic hydrostatic pressure and $s=$ deviatoric shear stress.

In SPH simulation, different constitutive models can be implemented to represent different material behaviors. In this study, the well-known Dracker-Prager constitutive model with non-associated flow rule has been introduced to SPH formulation to model the behavior of non-cohesive granular materials (Bui et al. 2008), hence, the stress rate is calculated:

$$\dot{\sigma}^{\alpha\beta} = \frac{D\sigma^{\alpha\beta}}{Dt} = 2G\dot{\varepsilon}^{\alpha\beta} + \left(K - \frac{2G}{3}\right)(\dot{\varepsilon}^{xx} + \dot{\varepsilon}^{yy} + \dot{\varepsilon}^{zz})\delta^{\alpha\beta} - \dot{\lambda}\frac{G}{\sqrt{J_2}}s^{\alpha\beta} \qquad (3)$$

Where, G and $K=$are shear and bulk modulus; $\dot{\varepsilon}^{\alpha\beta} = 1/2\left(\partial v^{\alpha}/\partial x^{\beta} + \partial v^{\beta}/\partial x^{\alpha}\right)$is total strain rate tensor consisting of the elastic and plastic strain rate tensor, and $\dot{\lambda}=$plastic multiplier change rate. More detail about SPH formulations and constitutive models can be found in previous works of the authors (e.g. Kermani and Qiu 2018b,c). The second order predictor-corrector scheme has been used to numerically integrate SPH differential equations

2.1.1 3D SPH model

Figure 1 shows a schematic representation of the 3D SPH cylindrical granular columns with initial aspect ratios (a) of 0.55 to 2.0, where $a = h_i/r_i$, initial height/initial radius. The columns consist of 1884-6280 SPH sand particles with typical properties of bulk modulus K =5 MPa, a Poisson ratio $\nu = 0.3$, a friction angle $\phi = 30°$, and a bulk density $\rho = 1540$ kg/cm^3 corresponding to specific gravity $G_s =$ 2.65 and a porosity n =0.42. The bottom surface is modeled using ghost boundary particles (Bui et al. 2008) while the cylindrical wall is modeled as dynamic boundary condition (Crespo et al. 2007). Comprehensive details about boundary treatments can be found in previous works of the authors (e.g. Kermani and Qiu 2017; Kermani and Qiu 2018b, c).

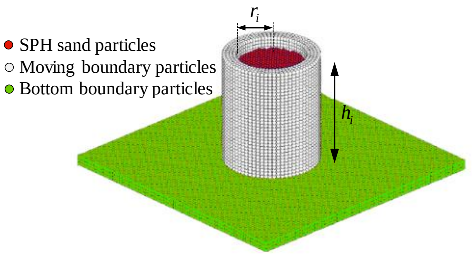

Figure 1. Schematic representation of a 3D SPH cylindrical column

2.2 Discrete Element Method (DEM)

DEM is a Lagrangian based computer simulation technique initially proposed by Cundall and Strack (1979), specifically suitable for simulation of granular materials. In this technique, the problem domain is represented by finite number of spherical particles within boundary walls, which follow Newton's equations of motion, and interact through different contact models (e.g. Hertz-Mindlin and Linear) (Itasca 2008; O'Sullivan 2011).

The hybrid approach proposed by Kermani et al. (2015) for determining contact properties is incorporated in the model in order to preserve the efficiency of simulations. In this hybrid approach, main simulations are conducted using linear contact models, but the contact stiffness parameters are estimated through initial simulations using the more sophisticated Hertz-Mindlin contact model.

Soil particles are modeled using spheres in DEM simulations, therefore for considering the combined effect of particle shapes and hysteresis behavior at contacts, "Rotational Resistance" is incorporated in the models, which directly reduces the rotational velocity of particles at each timestep by a constant factor of 0.95 (Kermani et al. 2015). This proposed method has been validated and proved to be simple and efficient (Kermani et al. 2015).

2.2.1 3D DEM model

In this study, 3D DEM models are made in the DEM software PFC3D® (Particle Flow Code in three dimensions, Itasca 2008). Figure 2 shows a schematic representation of 3D DEM cylindrical granular columns with initial aspect ratios ($a = h_i/r_i$) of 0.55 to 2.0 and porosity value (n) of 0.42., consisting of 10000-40000 DEM particles. In DEM simulation, due to its discrete nature, material properties of quartz at particle level has been implemented (i.e. density, ρ =2650 kg/cm^3, Poisson's ratio. $\nu = 0.3$, particle-particle friction angle, $\phi = 24^\circ$, and restitution parameter, $e = 0.75$). Details of generation of models can be found Kermani et al. 2014, 2015, 2018c).

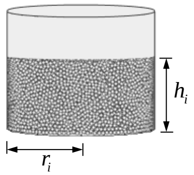

Figure 2. Schematic representation of a 3D DEM cylindrical column

The normal and tangential contact stiffness (k_n and k_s) for particles and walls for different aspect ratios are presented in Table 1.

Table 1. Equivalent k_n and k_s (N/m) for particles and walls for different a

$\boldsymbol{a}$	$\boldsymbol{k_{n(p)}}$	$\boldsymbol{k_{s(p)}}$	$\boldsymbol{k_{n(w)}}$	$\boldsymbol{k_{s(w)}}$
0.55	14500	12000	13000	11000
1.0	17000	14000	15000	12500
2.0	22000	18000	18000	15000

In order to model quasi-static axisymmetric collapse of granular columns, the cylindrical wall in DEM and SPH models is expanded with a radial velocity of u_r which should satisfy the quasi-static criteria (i.e., $\frac{u_r d_p}{r_i\sqrt{g_z h_i}}<10^{-3}$) where d_p =particle size, (Midi 2004).

3 RESULTS

The axisymmetric quasi-static collapse of granular columns with a =0.55, 1 and 2.0 has been simulated using 3D SPH and DEM models. The final deposit profile (e.g., final height and runout distance) and energy dissipation were studied using SPH and DEM method from continuum and discrete perspectives.

3.1 Collapse pattern

According to the results, different collapse patterns are observed depending on the initial aspect ratio in both models. For small aspect ratios (e.g., $a = 0.55$), only granular particles close to the moving wall slope down, leaving an undisturbed area at the center of column with flat surface at the top. The final profile looks like a truncated cone and it keeps its original height. For larger aspect ratios (e.g., $a = 2.0$), the entire column drops initially, keeping its flat surface at the time, then granular particles slope downs towards the moving wall. The final profile looks like a cone with a distinct tip. Figure 3 and 4 shows a qualitative comparison between DEM (black) and SPH (red) models for a =0.55 and 2.0, respectively. Only granular particles are shown in the figures and time is normalized based on the final time when the final deposit formed (t_f). As it can be observed, the 3D SPH and DEM models demonstrate a qualitatively consistent collapse pattern for quasi-static axisymmetric collapse.

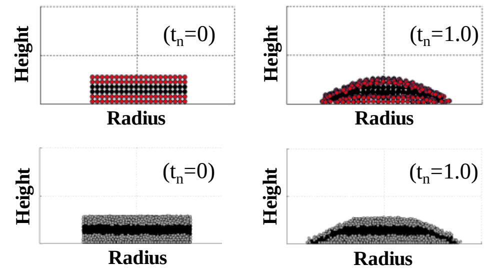

Figure 3. Initial column (at t_n =0) and final deposit profile (at t_n =1.0) for a column with a =0.55; SPH (red/black) and DEM (gray/black)

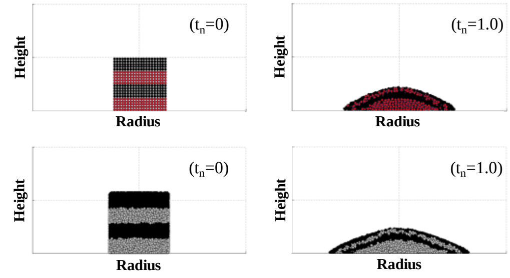

Figure 4. Initial column (at t_n =0) and final deposit profile (at t_n =1.0) for a column with a =2.0; SPH (red/black) and DEM (gray/black)

3.2 Final deposit height and runout distance

In order to compare the results of 3D SPH and DEM models quantitatively, final height (h_f) and runout distance (r_f) were monitored and compared. Figure 5 shows the normalized final height and runout distance for a =0.55, 1 and 2.0 for the 3D SPH and DEM models.

The results of DEM and SPH simulations are generally consistent in terms of final height and runout distance, with DEM simulations predicting slightly larger runout distances for different aspect ratios.

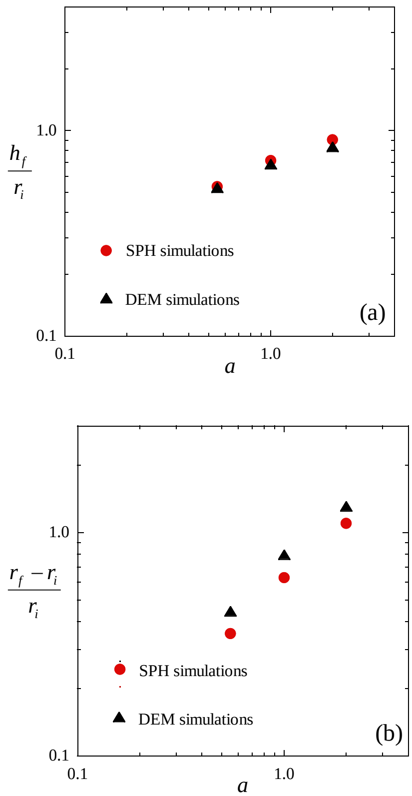

Figure 5. Comparison of SPH simulations, DEM simulations (a) normalized final height; (b)

3.3 *Evolution of final height and final runout distance*

Figure 6 shows the evolution of normalized height h/h_i at the center and corner of deposit for a column with a =2.0. Figure 6 demonstrates consistency and good agreement between the DEM and SPH results during different stage of collapse. DEM results in slightly higher runout distance, this difference can be observed at the final stage of the flow, where the edge of deposit become thin. At these regions, the SPH prediction is affected due to its continuum nature, while DEM benefits from its discrete nature to capture particle-level behavior at those regions.

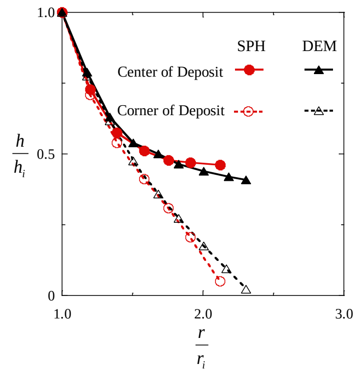

Figure 6. Evolution of normalized height h/h_i for a =2.0 from SPH and DEM simulations

3.4 *Energy dissipation*

Energy dissipation is an important quantity in study of the granular collapse. The Energy dissipation ΔE which occurs during the collapse is calculated as the difference between the initial potential energy E_{pi} and final potential energy E_{pf}, as kinetic energy is zero at initial and final stages of collapse. Potential Energies at different stages of collapse can be measured in SPH and DEM models knowing the height of each particle during the collapse. The normalized energy dissipation $\frac{\Delta E}{E_{pi}}$ for different aspect ratios are shown in Figure 7 based on SPH and DEM simulations. As it can be seen, (1) results of the two methods are in good agreement; and (2) normalized energy dissipation depends on initial aspect ratios and increases as aspect ratio increases.

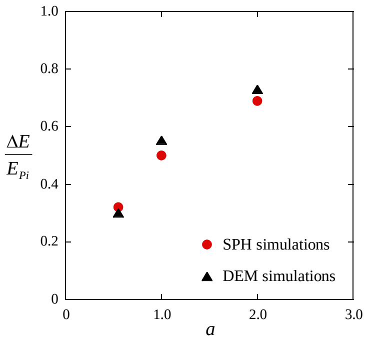

Figure 7. Normalized energy dissipation $\frac{\Delta E}{E_{pi}}$ *vs.* a

Energy dissipation during granular collapse is caused by dissipative particle-particle interactions which include frictional, damping and rotational resistance. Due to continuum nature of SPH method, it is not possible to estimate the breakdown of different components of energy dissipation. However, DEM method provides insightful information in this regard at micro-scale/particle level. Figure 7 shows the variation of frictional energy dissipation to total energy dissipation $\frac{E_f}{E_d}$ during the collapse. Figure 7 shows that the frictional energy dissipation increases steadily with time for all aspect ratios; and the frictional energy dissipation is more pronounced in smaller aspect ratios (i.e. a =0.55).

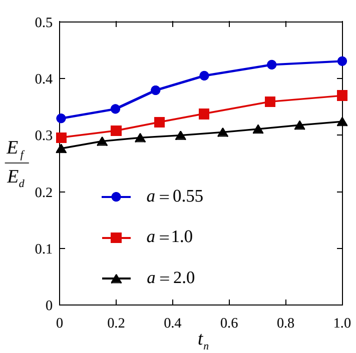

Figure 8. Normalized frictional energy dissipation $\frac{E_f}{E_d}$ *vs.* t_n *in DEM simulations*

4 CONCLUSIONS

In this study, the quasi-static axisymmetric collapse of granular columns is modeled using developed 3D DEM and SPH models and a comprehensive comparison between the capability of these two simulation techniques from discrete and continuum perspectives is presented. In SPH simulations, the granular materials are modeled as elastoplastic materials undergoing large deformation by using Drucker-Prager constitutive model with a non-associated plastic flow rule. In DEM simulation, rotational resistance factor was applied to reduce angular velocity of particles at each time step to implement the effects of particle shape and hysteretic contact behavior.

This study revealed that both SPH and DEM are capable of qualitatively and quantitively modeling quasi-static deformation of granular materials, which is observed during active movement of retaining structures. It was found that the results of SPH and DEM simulations are consistent in terms of collapse pattern, final deposit profile (i.e., final height and runout distance) and energy dissipation, qualitatively and quantitatively. However, where the particle-level behaviors dominate (e.g., at flow front, sharp edges of free surface, and tracing frictional loss at contact) DEM method is recommended.

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