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Obtaining fragility curves on levees subjected to flooding

Obtention de courbes de fragilité sur les digues sujettes aux inondations

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ABSTRACT: This paper illustrates a methodology to obtain fragility curves of a homogenous earth levee considering analytically two failure modes: a) slope stability, and b) internal erosion. The limit equilibrium method is applied for slope stability analyses and the finite element technique is used for the evaluation of internal erosion. The Monte Carlo simulation technique is used for the probabilistic model assumed for both failure modes through the specialized code SoilVision Systems Ltd (2014). Finally, the composite fragility curve for the studied levee is obtained using the fragility curves previously defined from each failure mode.

RÉSUMÉ: Dans cet article, la définition de la courbe de fragilité dans l'analyse de risque est définie. Ensuite, une méthodologie est proposée pour obtenir la courbe de fragilité d'une levée de terre homogène en considérant deux modes de défaillance: 1) la stabilité de la pente et 2) l'érosion interne. Dans l'évaluation de la stabilité de la pente, la méthode de l'équilibre limite est appliquée et la technique des éléments finis est utilisée pour étudier l'érosion interne. Le modèle probabiliste supposé pour les deux modes de défaillance est la méthode de simulation de Monte Carlo utilisant le code spécialisé SoilVision Systems Ltd (2014). Une fois que les courbes de fragilité de chaque mode de défaillance sont obtenues, la courbe de fragilité composite de la levée est obtenue, en combinant les courbes initiales de chaque mode.

Keywords: levee, fragility, Montecarlo, probability, flood

1 INTRODUCTION

Levees are raised structures (also call dikes, digues or flood defence embankments), predominantly of earth materials, whose primary aim is to provide protection against fluvial and coastal flood events along coasts, rivers and artifical waterways.

Levees are arrangements of components that provide individual adaptations to several types of loads. Figure 1 shows all the different components that may be found in a levee (Sharp *et al.*, 2013). In practice, not all of them are necessary and their inclusion depends on the specific purpose of the structure.

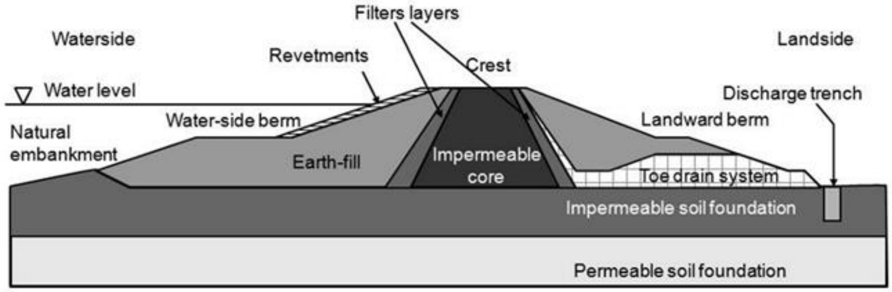

Figure 1. Individual components of a levee

Levee failure can occur by different failure modes, see International Levee Handbook, ILH (Sharp *et al.*, 2013). The three most common failure modes are: (a) external erosion, (b) internal erosion and (c) slope instability. In this paper, the last two modes are analytically addressed.

Despite their apparent simplicity, levees can be surprisingly complex structures. They are usually constructed by placing local fill material onto alluvial flood plains (with all their inherent natural variability). Unlike other engineering structures, levees can be irregular due to the nature and characteristics of their construction process and can markedly deteriorate over time if they are not well maintained.

In general, levee risk assessment does not include geotechnical reliability analyses. In fact, the geotechnical analyses for this type of structures has a deterministic instead of a probabilistic approach. The complexity of the system requires a clear method to show the influence of the different loads on the calculated probability of failure. The fragility curves provide such method and help understanding the levee behaviour. Fragility curves can also be used to update probability of failure by using information of survived loads, see Schweckendiek *et al.* (2017).

This paper illustrates a methodology to derive a fragility curve of a homogenous earth levee. The paper shows how probability of failure for different failure modes can be combined to a single fragility curve, indicating how the probability of failure increases as the water load is rising.

2 PROPOSED METHODOLOGY

2.1 The conditional probability-of-failure function

In this study, the term *failure* is defined as the unintended flooding of the protected area, and thus it includes both overtopping and breaching of the levee at water elevations below the crown. For an existing levee subjected to a flood, the probability of failure P_F can be expressed as a function of the flood water elevation (*FWE*) and other factors, such as soil strengh, permeability, embankment geometry and foundation stratigraphy. The analysis of levee reliability starts assuming the development of a conditional probability-of-failure function given the *FWE*. This can be established using engineering estimates of the probability functions or moments of the other relevant variables.

The conditional probability of failure $P_F(y)$ can be written as:

$$P_F(y) = P_F(failure|FWE) = f(FWE, X_1, X_2 \dots X_n) \quad (1)$$

where X_1 through X_n denote relevant parameters for the model (e.g., soil strength, hydraulic conductivity, upper stratum thickness). Equation

(1) states that the probability of failure $P_F(y)$ is a function of the intensity y (i.e., FWE).

The reliability R is defined as:

$$R(y) = 1 - P_F(y) \qquad (2)$$

Hence, for any flood water elevation, the addition of the probability of failure and reliability must be the unity.

The extreme values of $P_F(y)$ can be readily estimated by engineering judgment:

- For flood water elevation at the same level as the landside toe (bottom elevation), the levee is not loaded; hence, P_F = 0.
- For flood water elevation at or near the levee crown (upper elevation), $P_F \rightarrow 1$.

The question of primary interest, however, is the shape of the function between these extremes. For the case of flood water in the middle part of a levee, P_F could vary between zero and the unity, depending on factors such as levee geometry, soil strength, hydraulic conductivity, foundation stratigraphy, etc. Figure 2 illustrates four possible shapes of the P_F and R functions (Wolff, 2008). For a well-designed and constructed levee ("good levee"), the probability of failure P_F must remain low (high reliability) until the flood water elevation is rather high. In contrast, the P_F of a "poor levee" is high even when it is subjected to small flood heads. It is hypothesized that real levees may follow an intermediate path, similar to the "good" case for small floods, and to the "poor" one for floods of significant height. Quantifying this shape is the focus of the procedures that follow.

2.2 *Building fragility curves*

Figure 3 presents a general flowchart to build fragility curves in the context of risk analysis. The details for its application to evaluate the probability of failure on homogeneous levees are further addressed in Section 3.

3 APPLICATION TO A CASE STUDY

3.1 *Description of the case study*

Figure 4 shows the schematic levee cross-section assumed for the analyses (Sanchez, 2013). It consists of a homogenous clayey silt levee resting on a silty sand stratum. The failure modes analytically considered in this study are: (a) slope stability (rotational failure), and (b) internal erosion of the foundation (groundwater seepage). A total of six water levels (equally distributed) were analyzed varying between 0.5 to 5.5 m over the embankment. Beyond such levels, an existing levee will likely fail and lead to an overtopping failure mode.

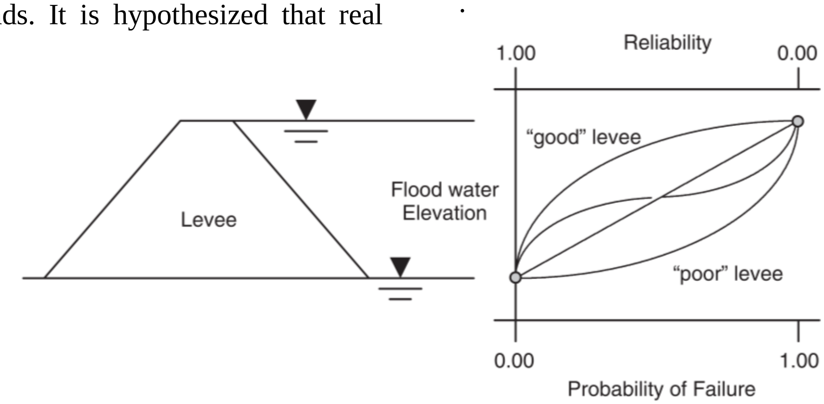

Figure 2. Possible probability of failure vs. flood water elevation functions

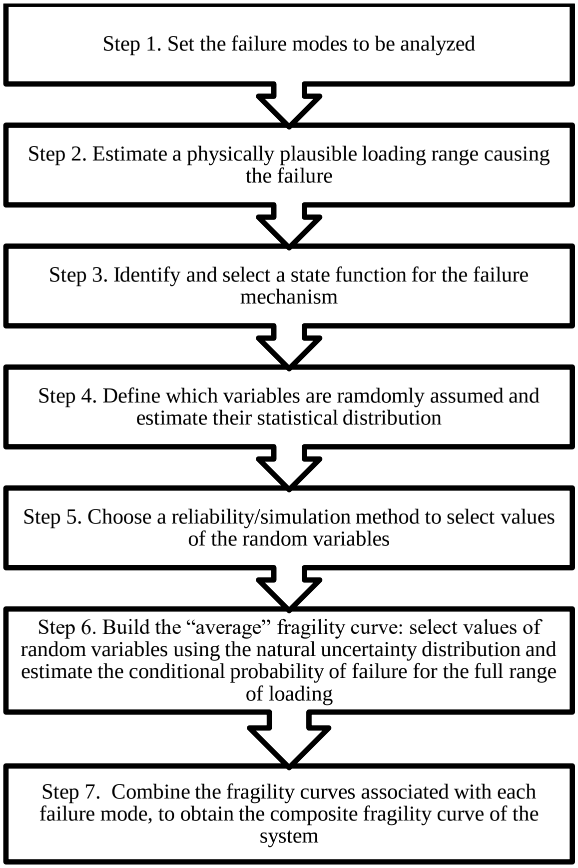

Figure 3. General flowchart to build fragility curves

The Montecarlo method is applied to estimate the probability distribution function, mean value $E[\cdot]$ and standard deviation σ of the factor of safety FS and the exit hydraulic gradients i_e propagating the uncertainty of the random variables characterized by their density functions, mean values and standard deviations.

This method has the advantage of conceptual simplicity, however, it requires a large number of realizations (i.e., simulated values of the interest function) to determine the statistical response with sufficient approximation for practical purposes. A total of 2000 realizations were executed for each failure mode evaluated here.

Table 1 presents the mean values and coefficients of variation CV of the mechanical and hydraulic properties assumed in the analyses. The water retention curve and hydraulic conductivity function of each material were estimated using the Van Genuchten model through the SVFLUX computer program. The assumed values fall within the typical range for geomaterials according to Wolff (2008) and Calamak (2014). All the mechanical properties were asummed to be normally distributed, while the uncertainty of the hydraulic properties was modelled considering a log-normal distribution.

3.2 Failure mode due to slope instability

3.2.1 Deterministic model and performance function

For slope stability, the choice of a deterministic analysis model is straightforward. A conventional limit-equilibrium slope stability model is applied that yields a factor of safety defined in terms of the shear strength of the soil.

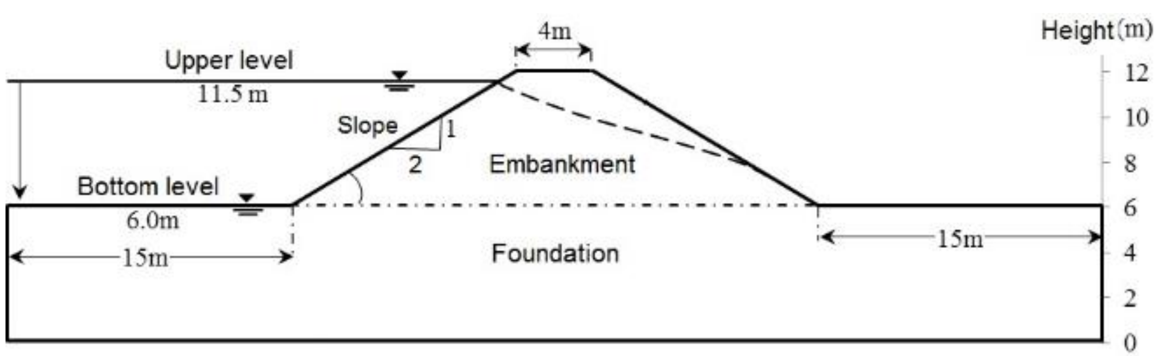

Figure 4. Cross-section assumed for slope instability and internal erosion example

Table 1. Mechanical and hydraulic properties of geomaterials

Parameter	Notation	Clayey silt (Embankment)		Silty sand (foundation)	
		Mean value	CV (%)	Mean value	CV (%)
Volumetric weight	γ	15 kN/m^3	-	17 kN/m^3	-
Internal friction angle	φ	10°	10	28°	8
Cohesion	c	12 kN/m^2	33	4 kN/m^2	50
Saturated permeability	k_s	1×10^{-6} m/s	400	1×10^{-4} m/s	200
Saturated volumetric water content	θ_s	0.4	-	0.32	-
Residual volumetric water content	θ_r	0.014	-	0.020	-
Fitting parameters	a	0.066	90	1.00	60
	m	0.44	-	0.76	-
	n	1.8	5	4.20	8

Depending on the probabilistic model being used, it may be sufficient to simply determine the factor of safety FS for various realizations of the random variables, and use these realizations to determine the probability that the factor of safety is less than one. For this failure mode, the performance function for slope stability can be considered as:

$$FS = 1 \tag{3}$$

For each *FWE*, the water surface whitin the embankment was obtained through a seepage analysis with the software SVFlux. Slope stability analyses were performed by Bishop's limit equilibrium method using the SVSLOPE program.

3.2.2 Random variables

The slope stability analyses study only the effects of the uncertainty in the mechanical properties of the geomaterials. Therefore, the hydraulic properties were supposed to be deterministic and equal to the mean values presented in Table 1.

3.2.3 Fragility curve

Figure 5 presents the relative frequency histogram of the *FS* for a water level *H* = 5.5 m. These data can be fitted to a normal probability distribution with $E[FS] = 1.332$ and $\sigma_{FS} = 0.189$ according to the Kolmogorov-Smirnov criterion.

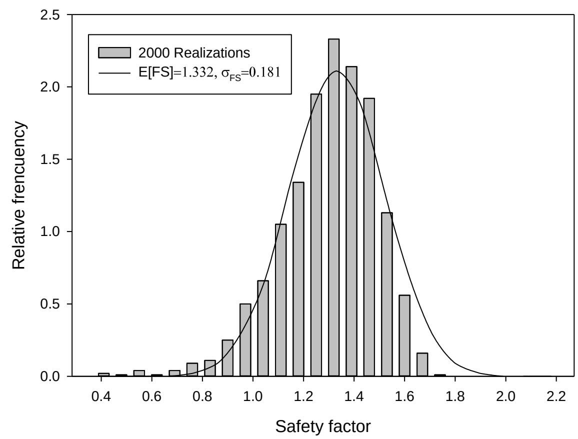

Figure 5. Histogram of relative frequencies of the factor of safety FS for H = 5.5 m

The reliability index β is defined as:

$$\beta = \frac{(E[FS]-1)}{\sigma_{FS}} \tag{4}$$

The reliability index β represents the number of standard deviations that separate the mean value of the safety factor from its defined failure value of 1. Therefore, it can be considered as a way of normalizing the safety factor with respect to its uncertainty. For the data presented in Figure 5 (H=5.5 m), β is equal to 1.75. The failure probability P_F of this case can be obtained by evaluating the standard normal cumulative distribution function in -β:

$$P_F(H = 5.5\ m) = 0.040 \tag{5}$$

Figure 6 shows the probabilities of failure of all the evaluated *FWE*. These results were fitted to a log-normal function with mean $E[ln(M)]$ and the standard deviation $\sigma_{ln(M)}$ equal to 6.209 and 2.372, respectively. In this figure a semilogarithm scale is used because of the low values of P_F. As expected, the fitted curve for this failure mode presents a good behavior since its shape is strictly increasing and convex, which is explained by the failure surface remaining within the embankment for all *FWE*. This means that the uncertainty in the *FS* mainly depends on the strength parameters of the embankment material and not on the foundation. For $H = 0.5\ m$, the influence of the water level is low, therefore the calculated failure probability mainly reflects the uncertainty on the strength parameters. This may explain the difference between $P_F(H = 0.5m)$ and the fitted curve.

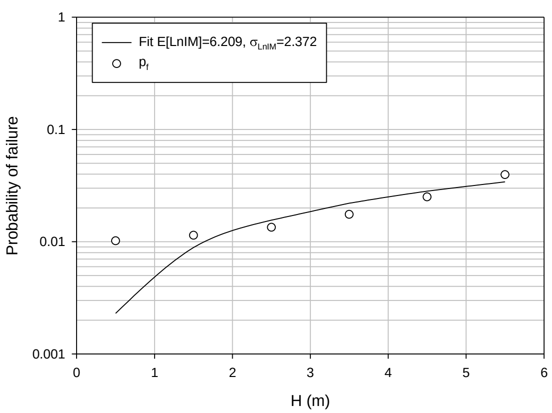

Figure 6. Probability of failure as a function of the intensity $y = H$ for failure mode of slope stability

3.3 Failure mode due to internal erosion

3.3.1 Deterministic model and performance function

For the internal erosion failure mode, failure is considered when the exit hydraulic gradient i_e equals the critical gradient i_c. Accordingly, the performance function is defined as:

$$\ln i_e = \ln i_c \qquad (6)$$

The critical gradient varies between 0.14 to 0.25 depending on the type of soil (Novak *et al.*, 2014). In this study, i_c was assumed to be 0.25. For each realization, a steady-state groundwater flow analysis was performed with the finite element software SVFlux. The exit hydraulic gradient was obtained from the gradient value in the Y direction at the toe of the slope.

3.3.2 Random variables

Groundwater flow analysis does not require the mechanical properties of the geomaterials. Additionally, SVFlux requires the fitting parameter *m*, the saturated and residual volumetric water content, which are assumed to be deterministic and equal to the mean values presented in Table 1. Thus, the random variables assumed for this failure mode were: the permeability and the fitting parameters α and *n*.

3.3.3 Fragility curve

Figure 7 shows the relative frequency histogram of the exit hydraulic gradient for *H* = 5.5 m. According to the Kolmogorov-Smirnov criterion, the data can be fitted to a log-normal distribution with $E[i_e]$ = 0.3066 and σ_{ie} = 0.0391. From the properties of the log-normal distribution, the standard deviation of $\ln i_e$ is 0.127 and its mean value is 1.190.

The probability of failure is then computed as:

$$P_F = P(\ln i_e > \ln 0.25) \qquad (7)$$

This probability can be estimated by first calculating the standard normalized variable *z*:

$$z = \frac{(\ln i_c - E[\ln i_e])}{\sigma_{lni_e}} = -1.544 \qquad (8)$$

For this value, the cumulative distribution function is *F*(*z*)=0.061, indicating the probability that the exit hydraulic gradient i_e is lower than the critical gradient i_c. The probability that the hydraulic gradient exceeds i_c is:

$$P_F = 1 - F(z) = 1 - 0.06142 = 0.9385 \qquad (9)$$

Note that the z value is analogous to the reliability index β, and it could be assumed as $\beta = -1.544$. The negative value for β follows from the calculated mean value i_e, for $H = 5.5\ m$, being larger than the critical gradient i_c and indicates a high probability of failure.

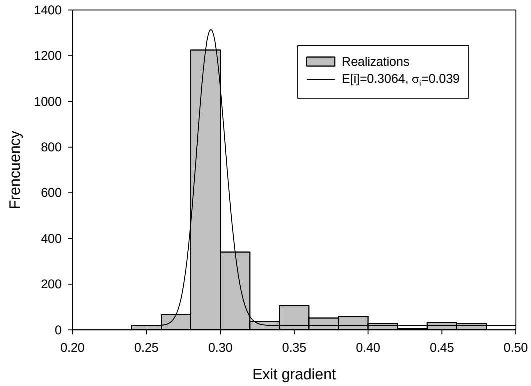

Figure 7. Histogram of relative frequencies of the exit hydraulic gradient for H = 5.5 m

The conditional probability of failure function can be obtained repeating this procedure for the others *FWE*, as shown in Figure 8. The data can be fairly fitted to a log-normal function with mean $E[ln(M)]$ and the standard deviation $\sigma_{ln(M)}$ equal to 1.468 and 0.146, respectively. Figure 8 shows that the probability of failure remains low for *FWE* less than 3.5 m, after which it has a convex shape until reaching a maximum of 93% when the water level is very close to the top of the levee.

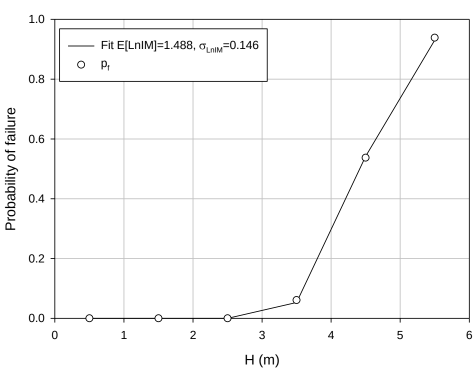

Figure 8. Probability of failure as a function of the intensity $y = H$ for failure mode of internal erosion

3.4 Combining failure modes

According to Wolff (2008), the estimation of probability of failure of an earth levee for multiple failure modes $P_{FC}(y)$ can be derived by combining the probabilities of failure P_F derived for individual failures modes as:

$$P_{FC}(y) = 1 - \prod_{i=1}^{k}(1 - P_{Fi}) \tag{10}$$

Equation (10) implies that the failure modes are independent. In the practice, this is a conservative estimation of P_{FC}.

A realistic $P_{FC}(y)$ should consider other circumstances which are not specifically addressed in the analytical models (e.g., animal burrows, cracks, roots, and poor maintenance). Thus, the empirical conditional probability of failure function proposed by Wolff (2008) (Table 2) was also included in the analysis.

Table 2. Empirical conditional probability of failure function (Wolff, 2008)

H(m)	0.5	1.5	2.5	3.5	4.5	5.5
P_F	0.00	0.01	0.02	0.20	0.40	0.80

The $P_{FC}(y)$ computed with Eq. (10) is shown in Figure 9. It is observed that probabilities of failure are generally quite low for *FWE* less than half the levee height and increase sharply as water levels approach the levee crest. Although there is insufficient data to judge whether this shape is a general trend for all levees, it has some basis in experience and intuition.

Figure 9 also shows that the fragility curve for slope stability is relatively flat, compared to the fragility for internal erosion, beyond $H = 2.5\ m$. This shows that the influence of the water level on the probability of slope failure is relatively small and uncertainty in strength parameters might have a larger influence on the probability of slope failure than the water level.

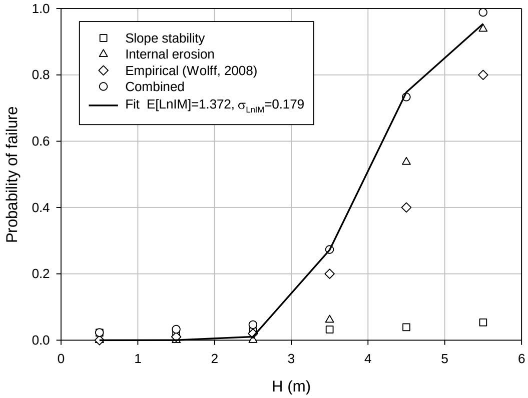

Figure 9. Probability of failure of an earth levee for multiple failure modes

4 CONCLUSIONS

The proposed methodology allows a more realistic analysis of a levee's failure. For the evaluated case, it is shown that the probability of failure strongly increases when the water level exceeds H = 2.5 m, due to internal erosion. The water level has a limited influence on the probability of slope failure. By involving the uncertainty in input parameters, geotechnical properties, it is possible to obtain the uncertainty in output parameters, such as the factor of safety, exit hydraulic gradient, etc. The interpretation of these results may determine a possible system failure.

For a more efficient application of the proposed methodology, it is recommended to perform a sensitivity analysis of the geotechnical parameters. Once this has been done, the parameters that have a greater influence on the considered failure mode can specifically be selected, reducing the time-consuming process.

For future research it is suggested to explore the use of other probabilistic techniques such as the point estimation method, the first order-second moments method (López-Acosta & Auvinet, 2011). Additionally, the convenience of using non-Gaussian functions to represent the probability density function of the input parameters should be evaluated. Finally, it would also be interesting to analyze the effect of uncertainty on other input parameters, such as the geometry of the levee, inclination of the slope, thickness of the foundation.

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