

Experimental investigation of granular flow erosion via photoelastic method

Etude expérimentale de l'érosion par écoulement granulaire via une méthode photoélastique

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ABSTRACT: Debris flows are particularly destructive and unpredictable high speed landslides which can grow in size by entraining material from the bed into their moving mass. The mechanism of bed erosion is still poorly understood however, with different evidences pointing to erosion via particle friction and collisions at the unsaturated head of the flow or erosion via loss of effective stress in the fluidized tail. This paper describes a new two dimensional physical model in which the impact and shearing of rapid dry granular materials is investigated via a photoelastic method aiming at understanding the fundamental mechanisms behind these processes. Disks of photoelastic material of different particle size are arranged within an erodible bed within a two-dimensional slope while a mass of similar sized disks are set to flow as a surge over the top. The stresses with the bed and within the flow are determined. The evolution of the stress profile and particle contacts with the passage of the flow and entrainment of bed materials is examined.

RÉSUMÉ: Les coulées de débris sont des glissements de terrain à grande vitesse particulièrement destructeurs et imprévisibles. Ils peuvent grossir en entraînant des matériaux du lit dans leur masse en mouvement. Le mécanisme de l'érosion du lit est toutefois encore mal compris, avec des évidences différentes indiquant une érosion via le frottement des particules et des collisions à la tête non saturée du flux ou l'érosion via une perte de contrainte effective dans la queue fluidisée. Cet article décrit un nouveau modèle physique bidimensionnel dans lequel l'impact et le cisaillement de matériaux granulaires à séchage rapide sont étudiés via une méthode photoélastique visant à comprendre les mécanismes fondamentaux de ce processus. Des disques de matériau photoélastique de différentes tailles de particules sont disposés dans un lit érodable dans une pente bidimensionnelle, tandis qu'une masse de disques de taille similaire est configurée pour s'écouler par-dessus. Les contraintes avec le lit et dans le flux sont déterminées. L'évolution du profil de contrainte et des contacts des particules avec le passage de l'écoulement et l'entraînement des matériaux du lit est examinée.

Keywords: granular materials; force chain; landslides; photoelasticity; bed-sediment entrainment

1 INTRODUCTION

In a granular material, forces are transmitted from one particle to the others via their contacts by an inhomogeneous network of force chains such that only a fraction of the total number of particles in a granular assembly carry the

majority of body forces and externally applied forces (Liu et al., 1995; Mueth et al., 1998; Majmudar and Behringer, 2005). When visualized through stress-induced birefringence within assemblies of photoelastic disks the complex force networks appear highly ramified and can undergo rapid changes in their

morphology throughout the material's loading history. The evolution of force chain microstructure is thought to be crucial to the development of bulk behaviour exhibited by granular materials but the relation linking the force chain architecture with their kinematics is not well understood yet. Force chains could also play a critical role in granular processes in geophysical flows such as snow and rock avalanches, mudflows and debris flows (Furbish et al., 2008; Estep and Dufek, 2012).

Debris flows are particularly destructive and unpredictable high speed landslides that can gain much of their mass and destructive power by entraining material from the bed into their moving mass. The mechanism of bed erosion is still poorly understood however, with different evidences pointing to erosion via particle friction and collisions at the unsaturated head of the flow or erosion via loss of effective stress in the fluidized tail (Iverson, 2012; Yohannes et al., 2012). In order to investigate how the transmission of the force chains at the base of geophysical flows can affect the complex phenomenon of bed-sediment entrainment we propose an experimental 2D physical model in which the impact and shearing of rapid dry granular materials is investigated via a photoelastic method.

2 EXPERIMENTAL METHODS

2.1 Apparatus and materials

During the experiments we generated two-dimensional gravity-driven granular flows in a small scale 2D flume made from Plexiglas sheets. The channel length is 1600mm and the width is approximately 8mm. 600mm before the end of channel, a section of 450mm long by 120mm deep has been cut out to accommodate the particles composing the erodible bed. At the top of the channel a tank stores the particles prior to the beginning of the tests (Figure 1). During the

tests discussed here, the slope of the channel was set at 30° to the horizontal.



Figure 1. Experimental apparatus

The granular material used for the experiments consists of 6 mm diameter, 1180 kg/m^3 density disks made from Vishay Precision Group's PSM-4 PhotoStress material. To obtain the photoelastic response of the particles, we illuminate the particles at the section where the disks in the channel bed were placed with a monochromatic green LED light. A circular polarizer film is mounted in front of the light panel and another one with orthogonal polaritation, on the face of the front plexiglass sidewall. Each experiment was recorded at 2000 fps by a Miro Phantom high-speed camera, equipped with a Nikor 50 mm lens. In this paper we describe the preliminary results obtained from the following configuration of materials.

The erosion bed was made using a bidisperse population of photoelastic disks, the majority of which were of 10mm diameter placed randomly together with 28 disks of 20mm diameter positioned so that they were not in touch with each other. The disks released from the tanks consisted of 400 disks with diameter of 10mm, 80 disks of 16mm diameter and 51 disks of 20mm diameter for a resulting area ratio of 2:1:1.

In the paper we describe the results obtained by the analysis of a series of 25 images sampled every 2.5 ms, recorded when the central part of the experimental granular flow overrode the erosion section.

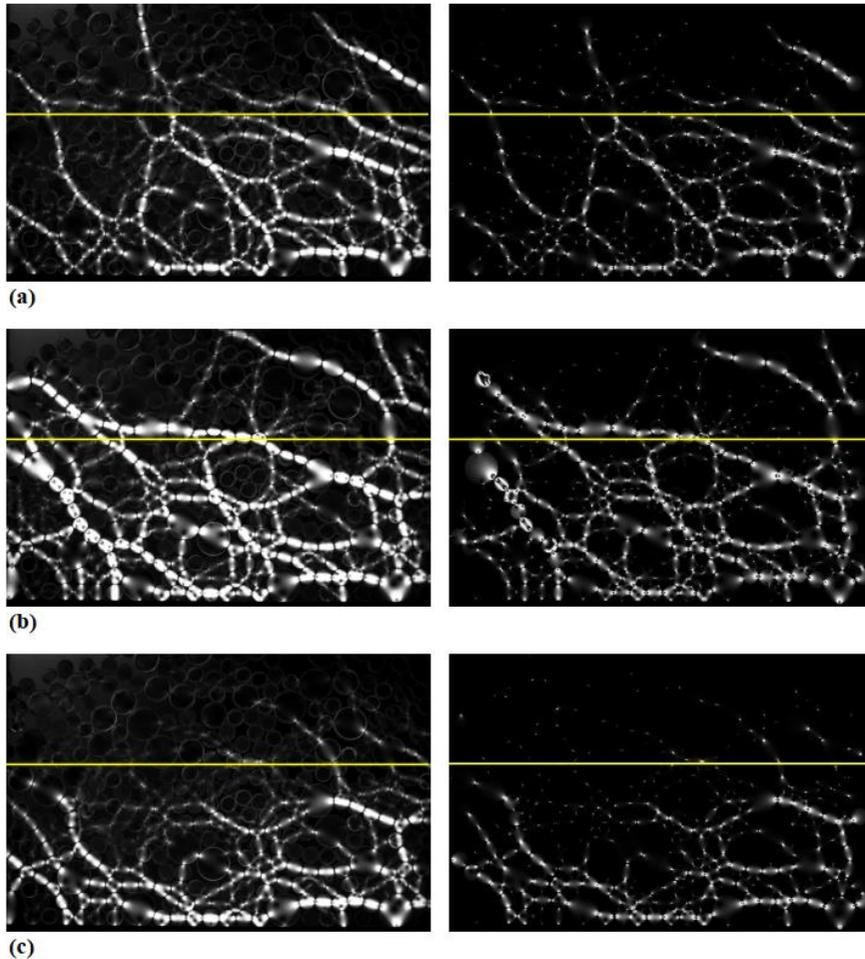


Figure 2. Sequence of images at time steps (a) $t=0$ ms, (b) $t=25$ ms and (c) $t=37.5$ ms. The left column shows the experimental images recorded by the high speed camera, the right column presents the synthetic images obtained using PeGS software (note that several disks closest to the boundaries in the synthetic images can exhibit numerical artifacts due to the lack of disk contacts at boundaries). The yellow line shows the top level of the erodible bed.

2.2 Force-chain extraction and characterization

In order to measure the vector contact forces on each particle we used the open source tool PeGS (Daniels et al., 2017) which is freely available on GitHub. PeGS software can estimate the contact forces as the result of the best fit of the experimental images with those obtained by adopting a theoretical model that exploits the

fact that photoelastic materials rotate the polarization of transmitted light by a known amount that depends on the local stress tensor.

As a result, we obtained two vectorial contact forces for each contact in the system since each disk is processed individually. We calibrated the software using the procedure described in Puckett (2012) and Majmudar (2006). The estimated average error was around 15-20%.

In order to characterise the force chain network in our granular material, we adopted the

community detection method, as proposed by Bassett et al. (2015). This approach is based on network science and it is particularly appropriate for particulate systems, where particles represent network nodes, and each interparticle contact is a network edge whose weight is the magnitude of the force at that contact (Papadopoulos et al., 2018). Following the procedure proposed in the aforementioned papers, we constructed a force-weighted contact network W from a list of all inter-particle forces setting $W_{ij} = f_{ij}/f_{avg}$ where f_{ij} is the normal force between particles i and j if they are in contact and f_{avg} is the mean force, and $W_{ij} = 0$ otherwise. Furthermore, $W_{ii} = 0$. Then we used the binary “adjacency matrix” B whose elements are

$$B_{ij} = \begin{cases} 1, & W_{ij} \neq 0, \\ 0, & W_{ij} = 0. \end{cases} \quad (1)$$

To obtain force chains from W , we determine communities of particles for which strong inter-particle forces occur. A well known method to identify communities in a network is by maximizing a quality function, known as modularity, with respect to the assignment of particles to sets called “communities”. Modularity Q is defined as:

$$Q = \sum_{i,j} [W_{ij} - \gamma P_{ij}] \delta(c_i, c_j) \quad (2)$$

where node i is assigned to community c_i , node j is assigned to community c_j , the Kronecker $\delta(c_i, c_j) = 1$ if and only if $c_i = c_j$, γ is the resolution parameter, and P_{ij} is the expected weight of the edge that connects node i and node j under a specified null model. The resolution parameter γ probes the organization of inter-particle forces across a range of spatial resolutions: small values of γ produce fewer large communities, while larger values of γ produce many smaller communities. The maximum value of modularity can be used to estimate the quality of a partition of a force network into sets of particles that are more

densely interconnected by strong forces than expected under a given null model.

An important choice in maximizing modularity optimization is the null model P_{ij} . For granular materials, particles can only be in contact with immediate neighbors and the so-called “geographical null model” is adopted to account for this constraint. The geographical null model is:

$$P_{ij} = \rho B_{ij} \quad (3)$$

where ρ is the mean edge weight in a network and B is the binary adjacency matrix of the network and, in our case, $\rho = 1$. For the geographical null model, maximizing Q assigns the particles into communities representing the force chains in a granular system.

Size, network force, and a gap factor characterize communities. The size s_c of a community c is the number of particles in that community. The network force of a community is:

$$\sigma_c = \sum_{i,j \in c} [W_{ij} - \gamma \rho B_{ij}] \quad (4)$$

where C is the set of nodes in community c . Finally, the gap factor g_c of a community c measures the presence of gaps and the extent of branching in a community. It is calculated as:

$$g_c = 1 - \frac{r_c s_c}{s_{max}} \quad (5)$$

where r_c is the value of the Pearson correlation between the upper triangle of the hop and Euclidian distance matrices of each community. For all the details we refer to Bassett et al. (2015)

The systemic values of the three diagnostics used in the paper are defined as the mean of s_c and σ_c over all communities and the weighted systemic gap factor as:

$$g_s = 1 - \frac{1}{n_{>1}} \sum_c \frac{r_c s_c}{s_{max}} \quad (6)$$

where the quantity n is the total number of communities, excluding singletons.

3 RESULTS

Figure 2 shows an example of successive images as recorded by the high speed camera at the investigated section. We defined $t=0$ ms as the time after 1.485s from the beginning of the release of the material from the top tank in the channel. The non uniform transmission of forces in the erodible bed at the base of the channel is clear and their propagation appears as a complex highly ramified network. For comparison also the synthetic images generated from the PeGS software are shown.

To quantify erosion during the test is not straightforward since the dynamic interactions between the particles in the erosion section and the overriding ones results in a mobilization of the particles in the bed that are pushed forward and upwards from their position and replaced by those incoming. Hence in order to quantify these interactions we analyze how the packing fraction of the erodible bed changes over time. The packing fraction is defined as:

$$\phi = A_d / A_{bed} \quad (7)$$

where A_d is the total area of the disks lying in the erodible bed and A_{bed} is the area of the erodible bed itself. Figure 3(a) shows how the packing fraction changes with time. Its value progressively decreases until time 17.5 ms, due, in particular to impacts between the disks entering the layer and the following movements of disks pushed in the flow direction. After that time, the packing fraction rapidly grows reaching its maximum values between 25 and 30 ms, with some incoming disks compressing the disks towards the bed of the erodible section. Afterwards, the parameter progressively decreases again.

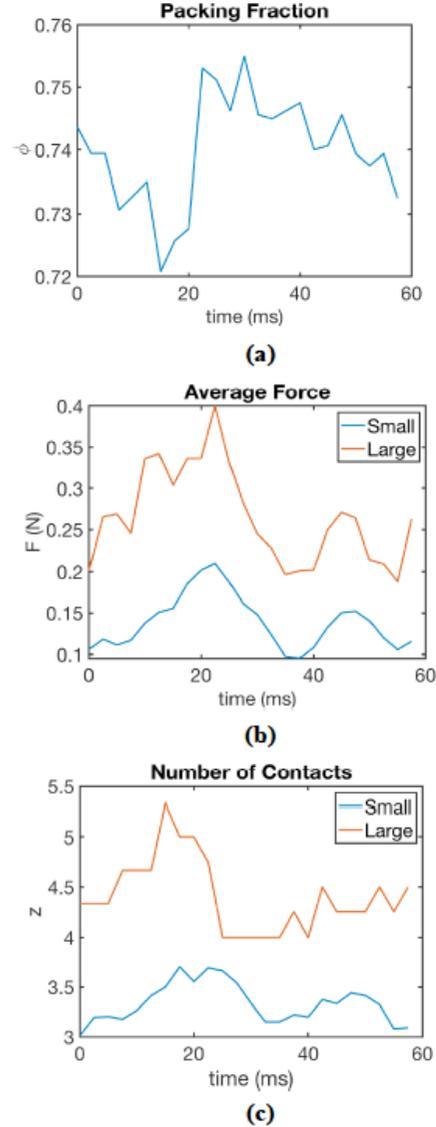


Figure 3. Estimates of the packing fraction (a), average total force on disks (b) and average number of contacts per particle (c) as a function of time

The mean total force of the disks is presented in panel (b) of Figure 3. Forces are calculated separately for the small and large disks. Forces on large particles are about twice larger than those experienced by small particles. However, the overall trend with time is similar for the two populations of disks. Force tends to grow from

time 0 to time 25 ms, where it reaches a maximum. Then it decreases for about 15 ms. Afterwards a similar pattern repeats, suggesting that average forces may follow a recursive behaviour. The average number of contacts per particle is shown in panel (c) of Figure 3. Its trend is similar to that of the forces, showing that the changes in the magnitude of the forces are related to a corresponding change in the number of contacts. However, the increase in the magnitude of the forces are also due to stronger interaction between particles, as demonstrated by calculating the ratio between the forces and the number of contacts (no figure is presented due to limited space).

The packing fraction and the force evolves differently in the first 17.5 ms. Although both the force and the number of contacts grows, the packing decreases. A visual inspection of the images suggests that the organization of the disk packing across the erodible bed is more effective in the presence of large stresses as characterized by highly branched force chains.

In order to describe the observed force chain patterns we analyze the results by means of the community detection methodology. We have focused on the three times $t=0$ ms, the first frame, $t=25$ ms at the force maximum and $t=37.5$ ms, when the force has a minimum. The resolution parameter γ has been optimized for the different disk distributions in order to extract the particle communities that are heterogeneous and branchlike ($\gamma=1.3$).

Figure 4 (a) shows in the left panel the communities identified at the initial time. Most of the communities have a linear and elongated shape. The region on the left of the erosion area presents few communities and most of the particles do not experience large forces except at the bottom, as confirmed from the images in the right panel where disks are colored by their resulting network forces.

Time 25 ms is shown in the central panels. A larger number of communities develops on the

left of the image, due to the increasing force in that region, as shown in the right panel. The strongest forces develops in the same area. Also big loops of particles are present, enclosing regions without meaningful contact forces.

At time 37.5 ms, the number of communities is similar to that at the beginning, as shown on the left panel of figure 4 (c). A large chain develops across all the erosion layers. Branches and loops have almost disappeared. Forces are everywhere weak except in the main chain, as shown in the right panel.

4 CONCLUSIONS

The paper describes the preliminary analyses of tests performed in an experimental 2D physical model in which the impact and shearing of rapid dry granular materials is investigated via a photoelastic method. The main goal of the proposed research is the study of the fundamental mechanisms behind the complex phenomenon of erosion in granular flows. Disks of photoelastic material of different particle sizes are arranged within an erodible bed within a two-dimensional slope while a mass of similar sized disks are set to flow as a surge over the top. Forces transmitted on the erosion layer are analysed at a microscale level, estimating forces at the contacts of each disk and at a mesoscale level, extracting and characterizing force chain structure using technique from community detection. The results show that the proposed methodology can be successfully adopted to characterize this kind of process. It will be applied to larger and more complete datasets in order to provide a framework for understanding which features of force chains are crucial to the study of erosion.

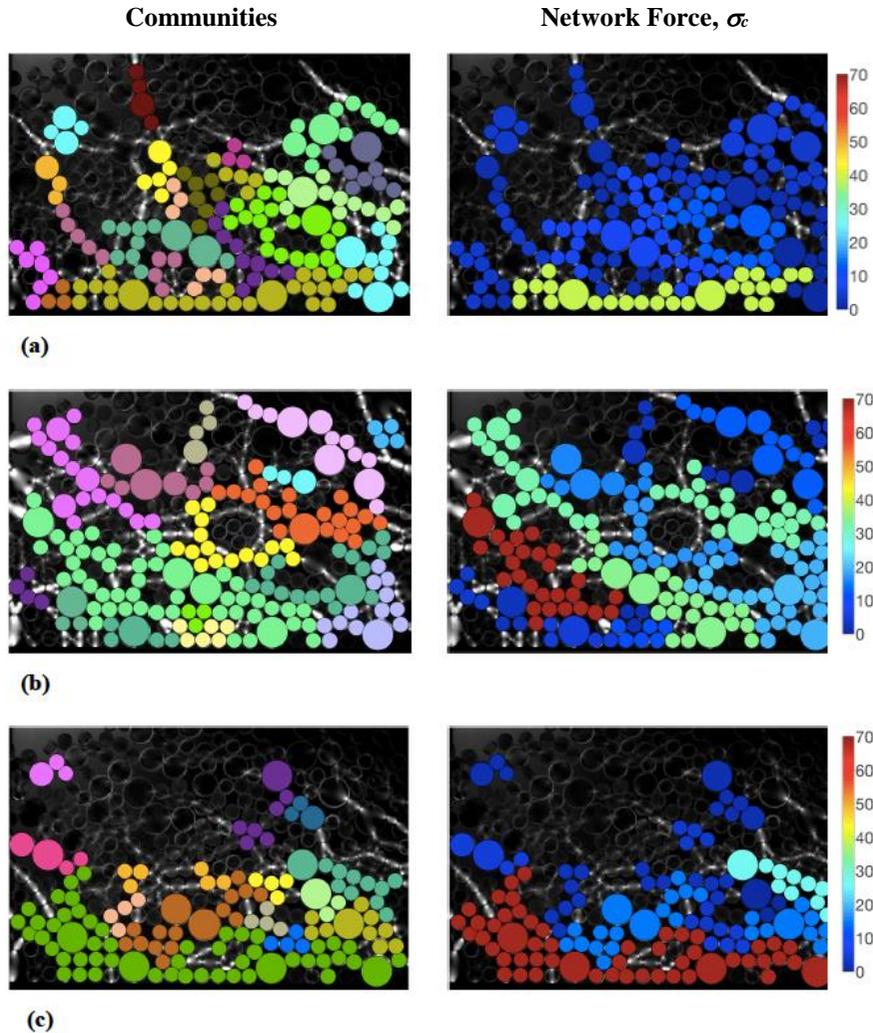


Figure 4. Sequence of images at time steps (a) $t=0$ ms, (b) $t=25$ ms and (c) $t=37.5$ ms. The disks in the images in the left column with the same color belong to the same community. Images on the right column show the network force σ_c (for each community). The colorbar refers to network forces.

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