Proceedings of the XVII ECSMGE-2019
Geotechnical Engineering foundation of the future
ISBN 978-0-7277-6067-8

© The authors and IGS: All rights reserved, 2019
doi: 10.32075/17ECSMGE-2019-0447

XVII European Conference
ECSMGE 2019
Reykjavik
on Soil Mechanics
and Geotechnical Engineering

The influence of the dilatancy on the ultimate bearing capacity of the rock mass

L'influence de la dilatance sur la capacité portante de la masse rocheuse

Ana Alencar
ETSI Caminos, Canales y Puertos, Polytechnic University of Madrid, Madrid, Spain

Svetlana Melentijevic
Faculty of Geology, University of Complutense, Madrid, Spain

Rubén Galindo
ETSI Caminos, Canales y Puertos, Polytechnic University of Madrid, Madrid, Spain

ABSTRACT: The influence of the angle of dilatancy in the ultimate bearing capacity of the rock mass is an underestimated topic. Due to the innumerable influential parameters in the mechanical behavior of the rock mass, a method or equation has not been defined to estimate in a standard way the value of the angle of dilatancy for the different types of rock masses. However, it is accepted that the value of the dilatancy angle ranges from minimum 0 ° to the maximum value equal to the instantaneous friction angle. On the other hand, the calculation of the ultimate bearing capacity of the rock mass is an important topic, being mainly focused on projects related to dams and civil works. This article presents a study about the influence of the dilatancy in the ultimate bearing capacity of the rock mass under the Hoek and Brown failure criterion, considering as limits the associative and the non-associative flow rule with the null value of dilatancy angle, for the analysis of a strip footing with self-weight and a circular footing without the self-weight. The numerical model was realized in the commercial program FLAC (Itasca). A total of 192 basic cases have been studied, resulting from the combination of four influential parameters in the ultimate bearing capacity (m_o, width of the footing, GSI and UCS). These cases were calculated under four different hypotheses: strip footing with self-weight and a circular footing without the self-weight, both with the associated and non-associative flow rule, resulting in a total of 768 cases studied.

RÉSUMÉ: L'influence de l'angle de dilatance sur la capacité portante de la masse rocheuse est un sujet sous-estimé. En raison des innombrables paramètres qui peuvent influencer dans le comportement mécanique de la masse rocheuse, une méthode ou une équation n'a pas été définie pour estimer de manière standard la valeur de l'angle de dilatance pour les différents types de masses rocheuses. Cependant, il est admis que la valeur de l'angle de dilatance est comprise entre, au moins, 0° et, au plus égal, à l'angle de friction instantané. D'autre part, le calcul de la capacité portante de la masse rocheuse est un sujet important, étant principalement axé sur des projets liés aux barrages et ouvrages de génie civil. Cet article présente une étude sur l'influence de la dilatance dans la capacité portante de la masse rocheuse selon le critère de rupture de Hoek et Brown, considérant comme limites la nullité et la dilatance associée, pour l'hypothèse d'une semelle filante, compte tenu du poids propre du terrain et une semelle circulaire sans tenir en compte le poids propre du terrain. Le modèle numérique a été réalisé dans le programme commercial FLAC (Itasca). Au total, 192 cas de base ont été étudiés, résultant de la combinaison de quatre paramètres influents dans la capacité portante (m_o, largeur de la semelle, GSI et RC). Ces cas ont été calculés dans quatre hypothèses différentes: semelle filante compte tenu du poids propre du terrain et semelle circulaire sans tenir en compte le poids propre du terrain, les deux avec la dilatance nulle et associée, résultant en un total de 768 cas étudiés.

Keywords: bearing capacity, dilatancy, shallow foundation, Hoek & Brown failure criterion, flow rule.

1 INTRODUCTION

The flow law is not an intrinsic hypothesis in the Hoek and Brown failure criterion, it should be adopted for the method chosen for the estimation of the bearing capacity. The published methods usually do not define which flow law is adopted, in some formulation, the associative flow rule is assumed, without knowing how that hypothesis influences the ultimate bearing capacity of rock mass. No technical literature was found that quantifies how the flow rule conditions the ultimate bearing capacity of the rock mass.

In general, in the everyday practice, the dilatancy is usually not taken into consideration in rock engineering. The papers related to the influence of the dilatancy angle or the flow law on the rock mass engineering problems are focused on the slope stability analysis and tunneling.

In the field of tunnels, Alejano and Alonso (2005) proposed a method for the estimation of the variation of dilatancy angle depending on the stress state, in a way that allows a better adjustment of the tunnel support (shotcrete, reinforcement) and the estimation of deformations during underground excavations.

Serrano et al. (2011) developed a convergence curves for circular tunnel subjected to hydrostatic pressures based on the theory of plasticity. The solution is valid for any failure criterion and for any plastic flow law, considering that the dilatancy varies from one point to another in the plastic area as the result of being the function of failure stresses and the instantaneous friction angle.

Melentijevic et al. (2017) emphasized that the hypothesis of the associative flow law is broadly accepted for its application in the slope stability analysis, resulting in the value of the factor of safety that is evidently higher than value obtained under the assumption of the non-associative flow law. Furthermore, the influence of the dilatancy angle on the value of factor of safety depends on the rock mass quality.

Considering that the mechanical behavior of the rock masses is non-linear, Detournay (1986) claimed that the assumption of a constant dilatancy angle is believed to be unrealistic because the dilation should be a function of the plastic strain (damage) and the confining stress.

Thus, the definition of the value the dilatancy angle is very complex, the adoption of null dilatancy ($\psi = 0^{o}$) or the associative flow rule ($\psi = \phi$) is usual. The adoption of the null dilatancy in some cases underestimates, but never overestimates the ultimate bearing capacity of the rock mass, so this option is on the conservative side.

Authors such as Hoek and Brown (1997), and Vermeer and Borst (1984) claim that the associative flow rule overestimates the strength of the rock mass, the value of the dilatancy angle recommended by these authors is much lower than the instantaneous friction angle. According to Vermeer and Borst (1984) the soil mechanics concept of a dilatancy angle is useful for concrete and rock, and the dilatancy angle is at least 20º less than the internal friction angle. Hoek and Brown (1997) proposed correlations depending on the overall geotechnical quality of the rock mass (geological strength index = GSI): (A) $\psi =$

$\phi/4$ for very good quality hard rock mass, that presents an elastic-brittle behavior (GSI=75); (B) $\psi = \phi/8$ for average quality rock mass, considering the behavior as strain softening (GSI=50); (C) $\psi = 0^o$ for very poor quality rock mass, that behaves perfectly plastically (GSI=25).

In this paper the study of the influence of the dilatancy angle on the ultimate bearing capacity of shallow foundations on rock mass is presented. This influence is analyzed considering two different hypotheses: (A) plane strain condition with the self-weight of the rock mass (strip footing); (B) axisymmetric model for the weightless rock mass (circular footing).

Merifield et al (2006) and Clausen (2013) have observed that for the poor quality rock (GSI < 30), the rock mass weight has a significant impact on the bearing capacity, whereas the self-weight has almost no effect for rocks of a higher quality. Independently of the influence of the self-weight rock mass on the bearing capacity, in this paper is studied the variation of the bearing capacity under the hypothesis of strip footing and self-weight rock mass, depending on the flow law adopted.

The second topic studied is also the influence of the flow law considering a circular footing and weightless rock material. According to Clausen (2013) no systematic results of bearing capacity of circular footings resting on a generalized Hoek–Brown rock material have previously been presented in the literature. The same author was proposed a bearing capacity factor, multiplying the uniaxial compressive strength (UCS) by this coefficient the bearing capacity of a circular footing on rock mass can be estimated. In this paper the influence of the flow law on the bearing capacity as function of the footing shape is presented.

2 NUMERICAL MODEL

A total of 192 basic cases have been analyzed, resulting from the combination of four influential parameters (m_o, width of the footing (B), uniaxial compressive strength of the rock (UCS) and geological strength index (GSI)) in the ultimate bearing capacity. With the values given in Table 1 a wide variety of types and states of the rock masses are covered.

Table 1. Summary of the parameters adopted

m_o	B (m)	UCS (MPa)	GSI
5 (claystone)	4.5	5	10
12 (gypsum)	11	10	50
20 (sandstone)	16.5	50	85
32 (granite)	22	100	

Numerical calculations were developed using 2-D models, applying the plane strain condition to represent a strip footing and the axisymmetry to simulate a circular footing. The last one uses a cylindrical coordinate system that allows to represent objects with axial symmetry, as is the case of a circular footing.

In the cases of the plane-strain condition a symmetrical model is used, where only half of the strip footing is represented. The axisymmetric grid is viewed as a unit-radian sector (FLAC, 2007). The boundaries of the both models are located at a distance that does not interfere in the result.

From the basic cases, four different calculation hypotheses are implemented (Table 2), resulting in a total of 768 cases. In all simulations the rough interface are adopetd and in the plane strain cases the unit weight of 26 kN/m^3 are used.

Table 2. Summary of the parameters adopted

Models type	Weight	Flow law
Plane-strain	Self-weight	Associative
Plane-strain	Self-weight	Non-associative (null dilatancy angle)
Axisymmetric	Weightless	Associative
Axisymmetric	Weightless	Non-associative (null dilatancy angle)

Numerically it is assumed that the ultimate bearing capacity is reached when the continuous medium does not admit more load, because an internal failure mechanism is formed. In FLAC the load is applied through velocity increments, and the ultimate bearing capacity is know from relation between stresses and displacements of one of the nodes (in this case the central node of the foundation is considered). A convergence study is carried out as well that consists in the analysis of values of the ultimate bearing capacity obtained under different increments of the velocity that is used.

3 THE INFLUENCE OF THE FLOW LAW CONSIDERING THE SELF-WEIGHT OF THE MATERIAL (STRIP FOUNDATION)

The comparison of the numerical results of the ultimate bearing capacity of the rock mass under different hypotheses of flow law is studied through a sensitivity analysis, where the influence of the geometrical and geotechnical parameters (m_o, B, UCS and GSI) is observed.

In Fig. 1 the correlation between the results with associative flow law (P_{hAS}) and non-associative flow law with null dilatancy angle (P_{hNS}) is observed, under the assumptions of strip footing and considering the self-weight of rock mass. From this figure can be concluded that the parameters that influence the correlation between P_{hNS} and P_{hAS} are GSI, m_o and UCS.

In Fig. 1-[A] and [C] can be observed that the difference between P_{hNS} and P_{hAS}, as well as the dispersion of the results, grow with the increase of mo and UCS. Being the results of P_{hAS} in the cases with lower m_o (m_o=5) similar to the results of P_{hNS} presenting a variation lower than 20%, while for higher m_o (m_o=32) P_{hAS} can exceed up to 70% the P_{hNS}. In relation to the UCS, the variation is lower, because for the range of lower values of the UCS= 5 - 10 MPa the dispersion is in the order of 0 - 40%, while the cases with UCS = 100 MPa present a range that varies between 0 and 65%.

In contrast, Fig. 1-[D] shows that with the low value of the GSI (GSI=10) the results varies up to 65%, and the decrease of the dispersion range for greater values of GSI is also observed.

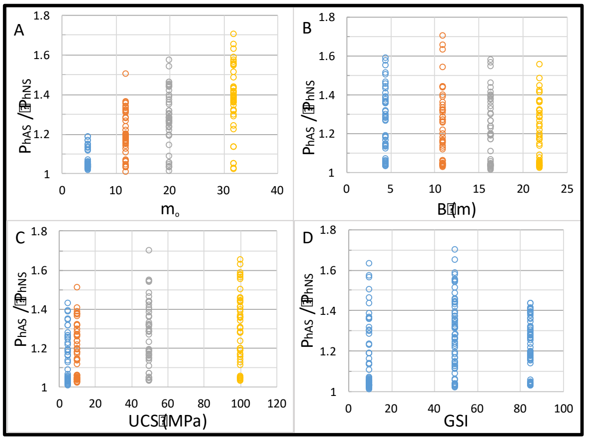

Figure 1. Correlation of P_{hAS} and P_{hNS} with 4 parameters (m_o, B, UCS and GSI).

From Fig. 1-[A] and Fig. 2-[A] can be concluded that the higher the value of m_o the type of flow law has a greater impact on the results.

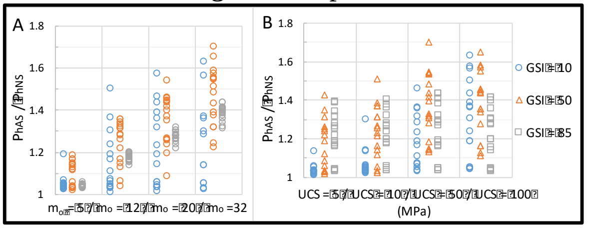

Figure 2. Relation between P_{hAS} and P_{hNS} for different GSI depending on m_o and UCS.

In Fig. 2-[B] it is observed that the influence of the UCS on the correlation between the results depends on the GSI. Concluding that the GSI is the dominant parameter between both, for greater values of the GSI (e.g. GSI = 85) the value of UCS has little effect on the dispersion of results.

Fig. 2-[B] shows that the dispersion range grow with the increase of the UCS, e.g. for UCS = 100 MPa the range varies between 5 to 65%. It is also observed that for the combination of low

GSI and UCS, the variation of the flow law type has very little effect on the correlation of results P_{hAS} and P_{hNS} (less than 10%). This is due to the fact that most part of the bearing capacity is associated to the self-weight of the rock mass.

The self-weight of the material is a confining load that displaces the Mohr circle to the right in the diagram representing the failure criterion. In addition, the inclination of the Hoek and Brown failure envelope depends directly on the value of the UCS (Fig. 3), for this reason in the cases of the low value of UCS, the envelope tends to get horizontal quickly (even for low stress state).

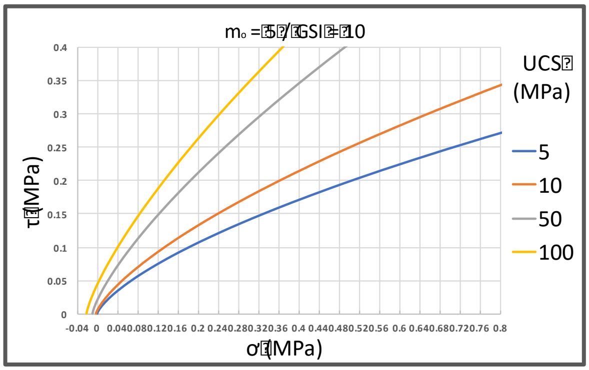

Figure 3. Representation of the Hoek and Brown failure envelope.

Thus, when the self-weight of the material is considered, in cases with low UCS and GSI, the Mohr circle is displaced to the right side, intercepts the Hoek and Brown failure envelope in the almost horizontal sector. In those cases, the value of the instantaneous internal friction angle is small, consequently, the value of the associative dilatancy angle as well. Therefore, the variation of the bearing capacity calculated with associative and non-associated (null dilatancy) flow law is very small, due to the fact that the value of the associative dilatancy angle is small.

4 THE INFLUENCE OF THE FLOW LAW IN A CIRCULAR FOUNDATION (WEIGHTLESS).

The circular footing shows a stress bulb different from the developed below a strip footing due to different shape. In this section, the influence of the flow law in cases of circular footing is studied, under the hypothesis of weightless rock mass and rough interface. Analyzing how the geometric and geotechnical parameters (m_o, B, UCS and GSI) influence the correlation between the bearing capacity estimated with associative flow law (P_{hAC}) and non-associative flow rule with null dilatancy (P_{hNC}).

4.1 Results of the analysis

Fig. 4 shows the correlation between P_{hAC} and P_{hNC} as function of each variable parameter, demonstrating clearly that mo and GSI are the two parameters that present most impact on the results correlation (P_{hAC} and P_{hNC}); once in Fig. 4-[A] and [D] it can be observed that the dispersion range depends on the value of the m_o and the GSI. In contrast, from Fig. 4-[B] and [C] can be concluded that B and UCS slight influence the bearing capacity obtained with different flow law.

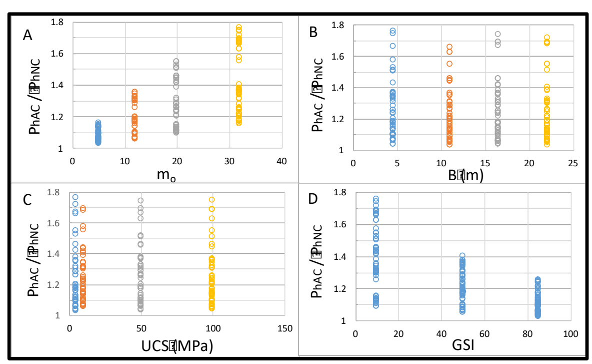

Figure 4. Correlation of P_{hAC} and P_{hNC} with the 4 parameters (m_o, B, UCS and GSI).

In Fig. 5 it can be observed the influence of the rock type (m_o) depending on other parameters (B, UCS and GSI). Confirming the trend observed in Fig. 4-[A] that the dispersion range increases with the increase of the m_o, e.g. for m_o = 5 the maximum variation between the results (P_{hAC} and P_{hNC}) is 20%, while for m_o = 32 the variation exceeds 70%.

Fig. 5-[A] and [B], in the same way of Fig. 4-[B] and [C], show that the B and the UCS do not influence the correlation of the results, due to the fact that the same range of results (P_{hAC}/P_{hNC}) are obtained for different values of B and UCS.

In relation to the GSI, it can be observed in Fig. 4-[D] that with the increase of the rock mass quality the influence of the flow law type on the bearing capacity reduces.

From Fig. 5-[C] can be concluded that for the combination of high values of GSI and low mo (e.g. GSI = 85 and m_o=5) the flow law used influences less than 10% the bearing capacity. On the other hand, in cases with low values of GSI and high values of mo (e.g. GSI = 10 and m_o=32) the flow rule is decisive on the bearing capacity result, P_{hAC} are at least 50% higher than P_{hNC}.

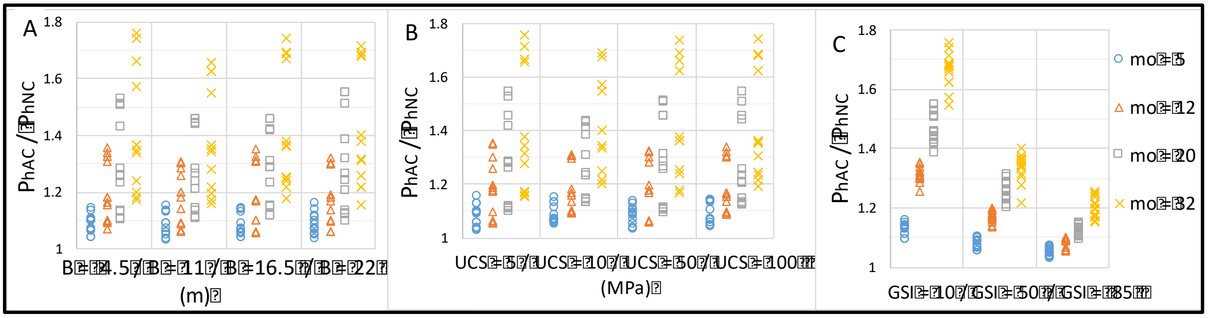

Figure 5 Relation between P_{hAC} and P_{hNC} for different m_o depending on B, UCS and GSI,

4.2 Displacements under the footing

In the numerical calculation, a stress path is formed until the failure is reached, considering the whole wedge of the ground below the footing. Therefore, different types of failure can be obtained, depending on the hypothesis adopted.

The graphic outputs of the displacements (horizontal and vertical) developed below the foundation are presented in this paper to understand how the failure mechanisms affect the results. It is emphasized that the two most influential parameters are mo and GSI, from the graphic outputs of the displacements this is confirmed as well. The displacement amount (horizontal and vertical) increases with the increment of the m_o value, the cases that show more displacement are the cases with more variation between P_{hAC} and P_{hNC}. In addition, it is observed that for the model under the assumption of associative flow law the displacements are higher.

From Fig. 6 can be concluded that adopting the hypothesis of associative flow law the horizontal displacement under the footing reduces with the increase of mo; this decrease is stronger in cases of poor quality rock mass (e.g. GSI = 10). That means that especially in cases of low GSI with the increase of the mo the failure mechanism changes, because of that for m_o = 5 the results of P_{hAC} and P_{hNC} are very similar, while for m_o = 32 the variation is great.

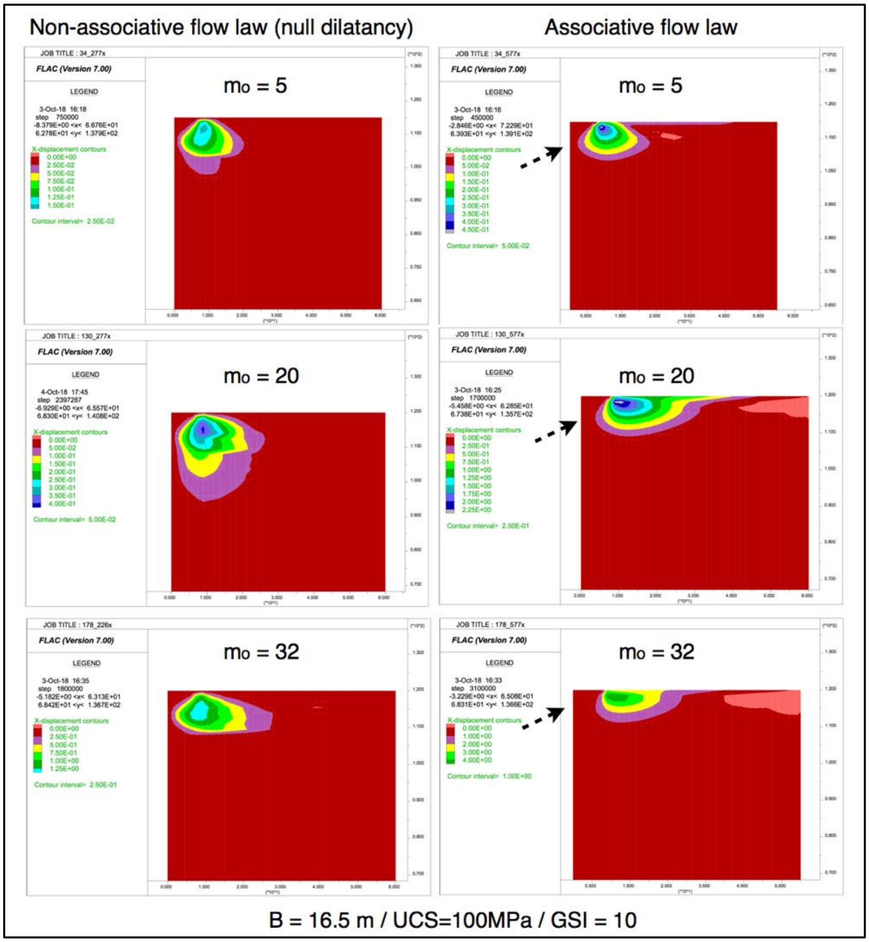

Figure 6. The variation of the horizontal displacements under the footing as a function of m_o.

In relation to the vertical displacement, the behavior is clearly different, with associative flow law the vertical displacement is concentrated in the lateral boundary of the footing, while with non-associative flow law with null dilatancy the vertical displacement under the footing is the most important.

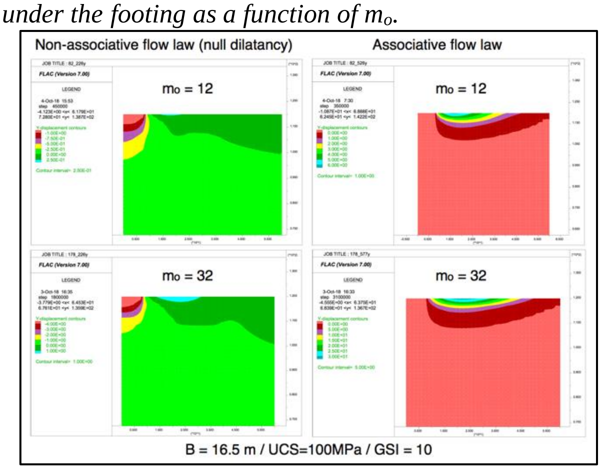

Figure 7. The variation of the vertical displacements under the footing as a function of m_o.

5 CONCLUSIONS

Taking into account the comparison described in previous sections for the cases studied under the hypothesis of the self-weight rock mass and the plain strain conditions (strip footing), the variation of the flow rule conditions the results of the bearing capacity in the following way:

- There are 3 parameters that strongly influence the correlation between the results of the bearing capacity obtained under hypothesis of the associative and the non-associative (null dilatancy) flow law: the rock type (m_o), the uniaxial compressive strength (UCS) and the geological strength index (GSI).
- The adoption of different flow law does not influence the low results of the bearing capacity (assuming low values of the GSI and the UCS), that differs from the cases of the weightless rock mass, in which the flow law precisely conditioned the low and medium results of bearing capacity.
- The foundation width (B) does not determine the correlation between the results, considering that it influences the results in the same way independently of the flow law type.

In general, the bearing capacity of the circular footing on the rock mass is influenced by the flow law adopted, based on the results obtained in 192 cases studied by numerical analysis through finite difference program FLAC, under the assumptions of weightless rock mass and rough interface, it can be concluded the following:

- The rock type (m_o) and the rock mass quality (GSI) are the parameters that most affect the correlation between (P_{hAC} and P_{hNC}).
- The cases with poor quality rock mass and high mo (e.g. GSI = 10 and m_o = 32) show greater dispersion range, with a variation between the results up to 70%.
- In contrast, with high values of GSI and low values of m_o (e.g. GSI = 85 and m_o = 5) the flow law adopted shows a small influence on the bearing capacity result.
- In relation to the footing width (B) and the UCS, it was not observed a constant effect of the value of these parameters on the ultimate bearing capacity of circular footing on the rock mass.

6 REFERENCES

Alejano, L. Alonso, E. 2005. Considerations of the dilatancy angle in rocks and rock masses. International Journal of Rock Mechanics and Mining Sciences. 42. 481-507. 10.1016/j.ijrmms.2005.01.003.

Serrano, A. Olalla, C. Reig, I. 2011. Convergence of circular tunnels in elastoplastic rock masses with non-linear failure criteria and non-associated flow laws. International Journal of Rock Mechanics and Mining Sciences - INT J ROCK MECH MINING SCI. 48. 878-887. 10.1016/j.ijrmms.2011.06.008.

Melentijevic, S., Serrano, A., Olalla, C., Galindo, R.A. 2017. Incorporation of non-associative flow rules into rock slope stability analysis. International Journal of Rock Mechanics and Mining Sciences. 96. 47-57. 10.1016/j.ijrmms.2017.04.010

Detournay, E. 1986. Elasto-plastic model of a deep tunnel for a rock with variable dilatancy. Rock Mech. Rock Eng.;19:99–108.

Hoek, E., Brown, E.T. 1997. Practical estimates of rock mass strength. Int J Min.;34(8):1165–1186.

Vermeer, P.A., de Borst R. 1984. Non associated plasticity for soils, concrete and rock. Heron;29(3):3–64.

Merifield, R.S., Lyamin, A.V., Sloan, S.W. 2006. Limit analysis solutions for the bearing capacity of rock masses using the generalised Hoek–Brown criterion. Int J Rock Mech Min Sci;43:920–37.

Clausen, J. 2013. Bearing Capacity of Circular Footings on a Hoek–Brown Material. In: International Journal of Rock Mechanics and Mining Sciences. Vol. 57. pp. 34-41.

FLAC. 2007. User's Manual, Itasca Consulting Group Inc. Minneapolis.