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A viscoelastic-viscoplastic, double yield surface constitutive model for fine-grained and organic soils

Modèle constitutif viscoélastique-viscoplastique à double surface de charge pour les sols à grains fins et les sols organiques

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ABSTRACT: Time and rate effects may have a crucial role in many engineering applications. In this work, an innovative viscoelastic and viscoplastic framework is presented within the framework of the overstress theory. The peculiarity of this approach is the use of two elastic and two plastic, rate-dependent mechanisms, having very different viscosity rates. This implies the use of two Perzyna-type, yield surfaces and the need of splitting the total strain increments in four fractions. Model simulations, obtained with a multiaxial formulation, are presented and compared with laboratory test results.

RÉSUMÉ: Les effets de temps et de la vitesse de déformation peuvent jouer un rôle crucial dans nombreuses applications d'ingénierie. Dans ce travail, un modèle innovant viscoélastique et viscoplastique est présenté dans le cadre de la théorie des contraintes effective. La particularité de cette approche réside dans l'utilisation de deux mécanismes élastiques et de deux mécanismes plastiques, dépendant de la vitesse, ayant des taux de viscosité très différents. Cela implique l'existence de deux surfaces de charge à la Perzyna et la nécessité de scinder les incréments de contrainte en quatre fractions. Les simulations du modèle, obtenues par une formulation multiaxiale, sont présentées et comparées aux résultats des tests de laboratoire.

Keywords: Soft clays, organic soils, constitutive modeling, visco-plasticity, two yield surfaces, flow rule.

1 INTRODUCTION

Time and rate effects assume different importance on the strength and deformation response of various geo-materials and may have a crucial role in many engineering applications. Experimental evidences show that the stress-strain-time response of fine-grained and organic soils are fairly similar to each other with minor, mainly quantitative, differences. In contrast, the response of coarse-grained soils is qualitatively different with respect to clay behavior (Lingaard et al. 2004).

The constitutive model proposed in this work belongs to the first class of viscous models proposed by Sekiguchi (1984), being based on the overstress theory. In particular, the model is the extension to multi-axial stress conditions of the viscoelastic-viscoplastic, 1D constitutive model proposed by Madaschi & Gajo (2015b, 2017).

The peculiarity of this approach is the identification of two elastic deformation mechanisms and two plastic, rate-dependent, mechanisms having very different viscosity rates. In particular, the short-term mechanism is assumed perfectly instantaneous for the elastic strains and low-viscous for the plastic response, whereas the long-term, elastic and plastic responses are both characterized by a high viscosity. The need for this approach cames from the phenomenological interpretation of 1D consolidation tests and undrained triaxial tests.

2 THE VISCO-PLASTIC MODEL

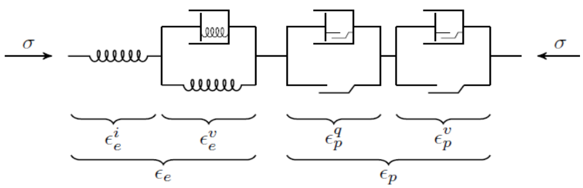

Figure 1. The proposed visco-elastic, visco-plastic, rehological model.

According to Fig. 1, the strain can be decomposed as follows

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_e + \boldsymbol{\epsilon}_p = \left(\boldsymbol{\epsilon}_e^i + \boldsymbol{\epsilon}_e^v\right) + \left(\boldsymbol{\epsilon}_p^q + \boldsymbol{\epsilon}_p^v\right) \qquad (1)$$

where the superscripts *i*, *q* and *v* stand for *instantaneous*, *quasi-instantaneous* and *viscous* (long-term), respectively.

The plastic behaviour is described by two Perzyna- type yield surfaces, having the shape of then modified Cam Clay model (Madaschi & Gajo, 2016). The long-term, *viscous* yield surface is defined as follows

$$\mathcal{F}_d^v\left(\boldsymbol{\sigma}, p_c^v, \mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_p^v\right) = \frac{3\,J_2}{(M\,k_\theta)^2} + \left(\frac{I_1}{3}\right)^2 + \frac{I_1}{3}\,p_c^v\,\Phi^v(\mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_p^v) = 0 \qquad (2)$$

where I_1 is the first invariant of the stress tensor, J_2 is the second invariant of the deviatoric stress tensor, M is the slope of the critical state line in compression, k_θ is a function describing the deviatoric section of the yield surface through the Lode angle θ, and $\Phi^v(\mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_p^v)$ is a suitable viscosity function (linear in the logarithm of the plastic volumetric strain rate):

$$\Phi^v\left(\mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_p^v\right) = \Gamma_p^v\, ln\left(\frac{\left|\mathrm{tr}\,\dot{\epsilon}_p^v\right|}{\dot{\epsilon}_{min}} + 1\right) \qquad (3)$$

In eq. (2), p_c^v represents the hardening parameter, defined as follows

$$p_c^v = p_0\, exp\left(\frac{\mathrm{tr}\,\boldsymbol{\epsilon}_p^v - \epsilon_{v0}}{(1-\alpha_p)\lambda}\right) \qquad (4)$$

where α_p is a suitable partition coefficient ranging between 0 and 1, such that for $\alpha_p = 0$, only the long-term, *viscous* yield surface is active. An equivalent formulation is assumed for the short-term, *quasi-instantaneous* yield surface (not reported here for the sake of brevity), which lays externally to the long-term, *viscous* yield surface.

The two Perzyna-type yield surfaces ($\mathcal{F}_d^v$ and $\mathcal{F}_d^q$) are associated with two static (inviscid) yield surfaces ($\mathcal{F}_s^v$ and $\mathcal{F}_s^q$) which are independent of the strain rate.

Depending on the position of the stress state with respect to the static yield surfaces, three cases are possible:

- when the stress state lays inside the *viscous* yield surface, the behaviour is visco-elastic (with elasticity law defined in the next Section);
- when the stress state lays between the *viscous* and the *quasi-instantaneous* yield surface, the behaviour is visco-elastic and visco-plastic and only the long-term, viscous mechanism of plastic deformation is active;
- when the stress state lays outside the *quasi-instantaneous* yield surface and the stress increment is directed outwards this surface, both plastic mechanisms are active, thus inducing both *quasi-instantaneous* and

viscous plastic deformations, in addition to viscoelastic deformations.

3 THE VISCO-ELASTIC MODEL

The assumed visco-elastic response is insipired to the inviscid Cam Clay elasticity, for the instantaneous fraction, whereas the delayed elastic behaviour is assumed to be ruled by the same viscosity function describing the visco-plastic response. The free energy density function is assumed to be given by the sum of two terms

$$\psi(\boldsymbol{\epsilon}_e, \boldsymbol{\epsilon}_e^v) = \psi_i(\boldsymbol{\epsilon}_e - \boldsymbol{\epsilon}_e^v) + \psi_v(\boldsymbol{\epsilon}_e^v) \tag{5}$$

where

$$\begin{aligned}\psi_i(\boldsymbol{\epsilon}_e^i) = \gamma_0(-\mathrm{tr}\,\boldsymbol{\epsilon}_e^i) + \\ + \sum_{k=1}^{7} \frac{\gamma_k}{(\alpha_e)^k}\left(-\mathrm{tr}\,\boldsymbol{\epsilon}_e^i\right)^{k+1} + \\ + \sum_{k=1}^{7} \frac{\beta_k}{(\alpha_e)^k}\left(\mathrm{tr}\,(\boldsymbol{\epsilon}_e^i)^2\right)^{(k+1)/2}\end{aligned} \tag{6}$$

with γ_k and β_k constitutive parameters. A similar expression is proposed for φ_v:

$$\begin{aligned}\psi_v(\boldsymbol{\epsilon}_e^v) = \gamma_0(-\mathrm{tr}\,\boldsymbol{\epsilon}_e^v) + \\ + \sum_{k=1}^{7} \frac{\gamma_k}{(1-\alpha_e)^k}(-\mathrm{tr}\,\boldsymbol{\epsilon}_e^v)^{k+1} + \\ + \sum_{k=1}^{7} \frac{\beta_k}{(1-\alpha_e)^k}(\mathrm{tr}\,(\boldsymbol{\epsilon}_e^v)^2)^{(k+1)/2}\end{aligned} \tag{7}$$

Note that the proposed expressions involve the summation of terms each one having the shape of the non-linear hyperelastic formuation proposed by Gajo and Bigoni (2008) and the constitutive parameters γ_k and β_k are selected with the aim of simulating a linear elastic response in a void-ratio versus logarithmic mean pressure plot, whereas the ratio beween γ_k and β_k depends on the assumed Poisson's ratio.

A suitable dissipation function is postulated for representing the viscoelastic behaviour

$$\phi(\boldsymbol{\epsilon}_e^v, \dot{\boldsymbol{\epsilon}}_e^v) = \left[\Gamma_e \log\left(\frac{|\mathrm{tr}\,\dot{\epsilon}_e^v|}{\dot{\epsilon}_{min}} + 1\right)\right] \mathrm{sgn}(\mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_e^v)\left(\frac{\partial \psi_v}{\partial \boldsymbol{\epsilon}_e^v} \cdot \dot{\boldsymbol{\epsilon}}_e^v\right) \tag{8}$$

where $\mathrm{tr}\,\boldsymbol{\epsilon}_e^v$ is the viscoelastic volumetric strain.

Throguh these equations, the visco-elastic relationships are deduced. It is worth emphasising that the viscoelastic strain rate is ruled by the rate of elastic volume strains $\mathrm{tr}\,\dot{\boldsymbol{\epsilon}}_e^v$. This is obviously a simplification, because the role of the rate of visco-elastic deviatoric strains is completely neglected. The effects of this assumption will be discussed in the next Sections.

4 MODEL SIMULATIONS OF OEDOMETRIC TESTS

The constitutive equations described in the previous Sections were implemented through a user-defined material subroutine (UMAT) in the commercial FEM code Abaqus (Hibbit et al., 2014). An implicit, backward-Euler integration scheme was used, associated with a line-search algorithm.

Madaschi and Gajo (1017) have recently proposed a 1D constitutive model based on the same principles described above, that was used for simulating oedometric tests performed on organic clays subjected to complex loading and unloading cycles. Notwithstanding the very good agreement between model simulations and experimental results, the correct simulation of oedometric tests with a 1D model may leave some doubts about the role of the horizontal stresses that are not taken into account by the model. This is why we started with the simulation of classical incremental loading oedometric tests performeded on a reconstituted organic clay.

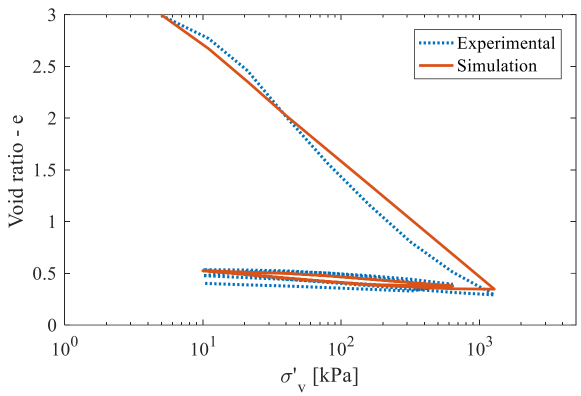

Figure 2. Comparison between the laboratory and the simulated oedometric compression curve.

Model simulations were obtained discretizing the oedometric sample with 10 quadratic finite elements for the solid skeleton, superposed to 10 linear elements for the pore pressures.

Figure 2 shows the comparison between experimental results and model simulations in terms of one dimensional oedometric compression curves.

It can be observed that model simulations capture very well also the unload-reload cycles with the associated histeresis, that is here mainly due to visco-elasticity. It is worth noting the linear reponse in the logarithimc plot of the unloading/reloading phases, that is properly simulated by the selected summation expression of the free energy density function.

The simulated evolution of the coefficient of earth pressure at rest K_0 is shown in Fig. 3. These values correspond to the condition at the end of consolidation, at which the distribution of K_0 along sample height is fairly uniform. It can be observed that, notwithstanding the unavailability of direct experimental measurements, the behaviour predicted by the model is reasonable with values close to 0.5 in the virgin loading and larger values in the unloading phases. Reload and unload phases follow distinct paths.

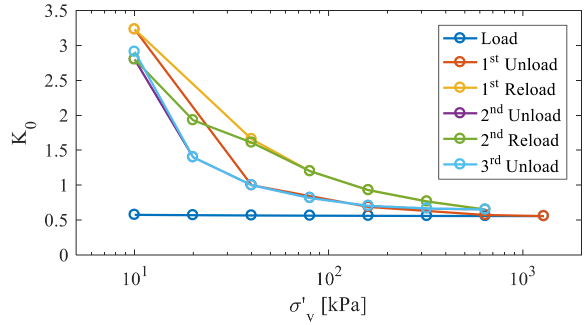

Figure 3. Evolution of the earth coefficient at rest.

Figure 4 shows the corresponding stress paths followed during oedometric consolidation in a *q-p'* plot. The different loading paths followed during unloading and reloading phases can be appreciated also in Fig. 4 (as in Fig. 3). These differences are almost entirely due to the visco-elastic behaviour and lead to reasonable model simulations.

The simulations are remarkly good considering that they were obained with the conventional Cam Clay yield surface, although associated with a Perzyna-type viscoplasticity.

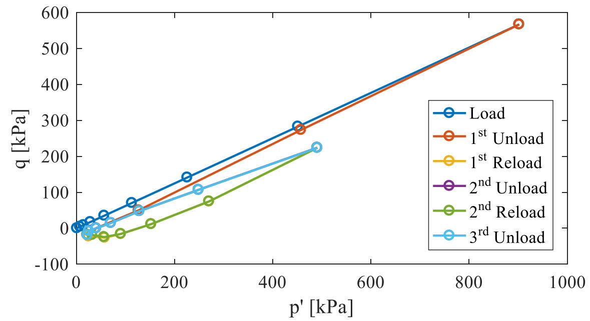

Figure 4. Stress-path followed during loading and unloading phases.

The comparison between the experimental results and simulations of primary and secondary conslidation in some representative oedometric steps are shown in Fig. 5. In particular, a virgin compression loading (from 320 to 640 kPa), an unloading (from 160 to 40 kPa) and finally a reloading (from 40 to 80 kPa) step have been considered. The corresponding dissipation of pore pressure as simulated by the model is shown

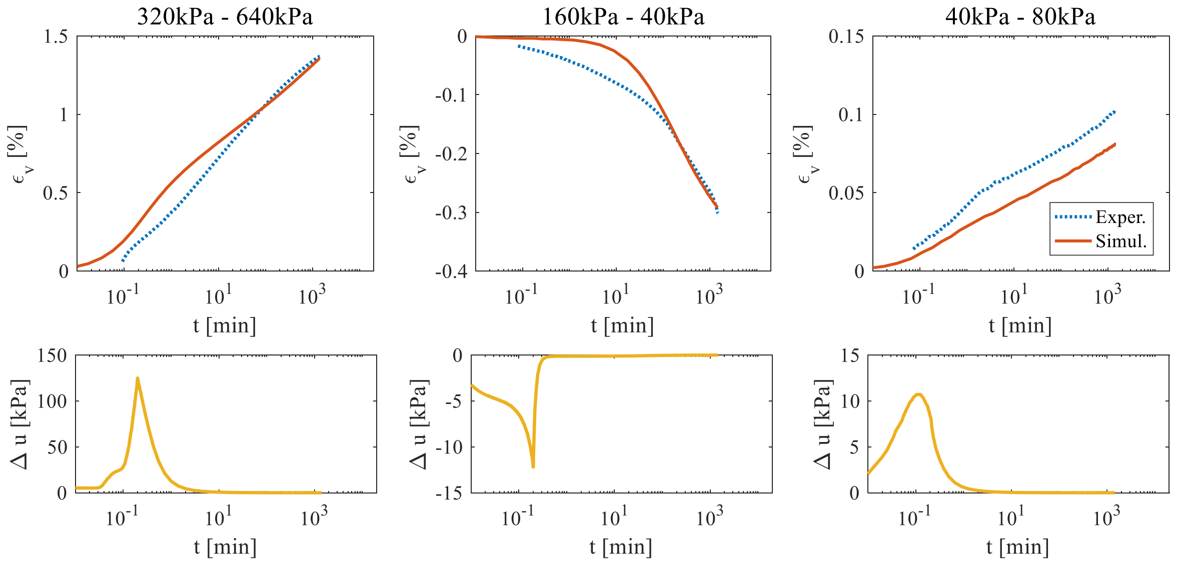

Figure 5. Comparison between experimental and simulated incremental loading steps of a conventional oedometric test on remoulded organic clay.

in Fig. 5. It is worth adding that in each loading/unloading step, the external load change was applied in 0.2 minutes, during which a part of pore pressure excess dissipated.

The permeability was estimated from the experimental results, thus the computed duration of the primary consolidation can be appreciated in Fig. 5. It can be observed that in most of the cases, the duration of the primary consolidation is smaller than 10 minutes and that the deformations due to both primary and secondary consolidation are correctly captured.

During the virgin loading phase (from 320 to 640 kPa) in Fig. 5, the initial pore pressure variation has a sort of wavy evolution, that is mainly related to the *quasi-instantaneous*, visco-plastic mechanism.

5 MODEL SIMULATION OF UNDRAINED SHEAR TESTS

Experimental results and model simulations of an undrained, compression test performed in a triaxial cell by Tatsuoka et al. (1998) is shown in Figs. 6. These results were obtained by changing the applied axial strain rate during the undrained compression test of a soft clay.

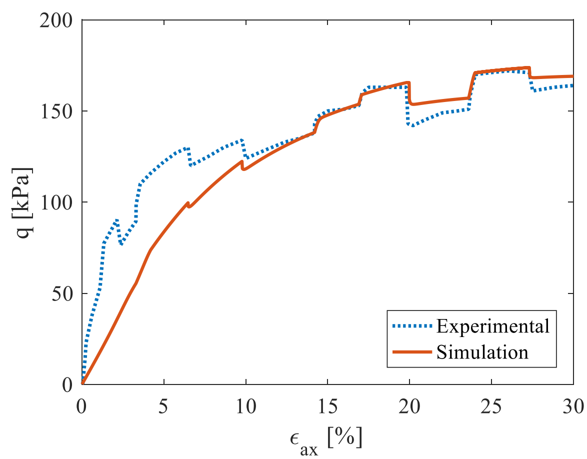

Figure 6. Stress-path followed during loading and unloading phases.

The consistency of model simulations with experimental results is particularly good at large strains, whereas the effects of the applied axial strain rate are underestimated at small axial strains. This is probably related to the assumption that the viscoelastic strain rate is ruled by the rate of elastic volume strains tr $\dot{\boldsymbol{\epsilon}}_e^v$, which is expected to be fairly small in the initial phases of the undrained loading (in fact the undrained effective stress path of a soft clay is initially almost vertical in a *q*-*p'* plot, with small increments of *p'*).

6 CONCLUSIONS

The constitutive model proposed in this work assumes two visco-plastic mechanisms having very different strain rates. This leads to a double yield surface approach, in which each yield surface is ruled by a Perzyna-type of hardening, with very different viscous response. The novelty of this work is the incorporation of a long-term, visco-elastic mechanism, beside the instantaneous elastic response. The viscoelastic response is assumed to follow the same response assumed for viscoplasticity, thus the delayed elastic response is mainly affected by the volumetric elastic strain rate.

The simulation of oedometric tests with complex unloading/reloading histories is very good, in terms of both compressibility and simulated evolution of horizontal stresses. Also the simulation of undrained triaxial compression tests subjected to sharp changes of the applied axial strain rate is very satisfactory. In the latter case however, some modifications are probably needed for improving the viscoelastic response.

The future developments of the model will be addressed to the simulation of heavily overconsolidated clays, for which some numerical difficulties still exist.

7 ACKNOWLEDGEMENTS

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