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Foundation Subgrade Reactions

Réactions du sol de Fondation

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ABSTRACT: In this article, the author strives to define an easily applicable method for estimating foundation subgrade reactions. The method is compared with Boussinesq's stress theory and by taking additional key parameters into account, the proposed method may lead to optimized foundation designs. The proposed method can be applied for designing foundations and estimating settlements. It has been based on Sadowsky's subgrade stress distribution and multiple FEM calculations. Validity of the method is discussed in terms of foundation rigidity and plastic behaviour of the subgrade.

RÉSUMÉ: Dans cet article, l'auteur s'efforce de définir une méthode aisément applicable pour estimer les réactions de fondations de fondation. La méthode est comparée à la théorie des contraintes de Boussinesq et, en prenant en compte d'autres paramètres clés, la méthode proposée peut permettre d'optimiser la conception. La méthode proposée peut être appliquée à la conception de fondations et à l'estimation de tassements. Il s'appuie sur la distribution des contraintes de la couche de base de Sadowsky et sur plusieurs calculs FEM. La validité de la méthode est discutée en termes de rigidité de fondation et de comportement plastique du support.

Keywords: Foundation design, Subgrade reactions, Settlements, Ballast numbers

1 INTRODUCTION

Subgrade reactions define stresses and settlements in subgrades in response to applied loads. The subgrade reactions are important when designing foundations as these have a large impact on settlements and cross-sectional forces in the foundations.

Subgrade reactions are quantified by the coefficient of subgrade reaction, that defines incremental stresses over incremental settlements as given by Eq. (1).

This article proposes a new method for calculating subgrade stresses, settlements and the coefficient of subgrade reaction.

$$K_s = \frac{\Delta p}{\Delta S} \qquad (1)$$

2 A NEW METHOD

The author of this article proposes Eq. (2) that defines the coefficient of subgrade reaction along the x-axis for a rigid strip foundation, see Fig. 1. The foundation is considered located onto an elastic medium with a Poisson's ratio of 0.3.

The equation is based on a merge of two individual methods for estimating the foundation subgrade stresses and settlements. The two method takes basis in the same assumptions, which allow them to merge.

In the following, derivation of the proposed equation is presented. That constitutes derivation of incremental stresses and incremental settlements, respectively.

$$K_S = \frac{M}{F_S \cdot \pi \cdot \sqrt{a^2 - x^2}} \qquad (2)$$

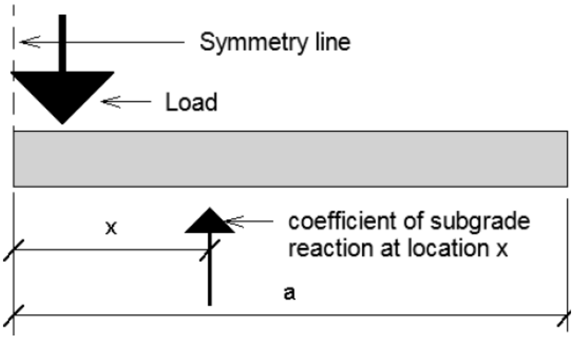

Figure 1. Cross section of a strip foundation

2.1 Subgrade Stresses

The vertical stress distribution is given by Eq. (3), credited Sadowsky [2]. The equation is statically admissible for a rigid foundation located onto an elastic subgrade with Poisson's ratio of 0.5.

$$\sigma_z = \frac{P}{\pi \cdot \sqrt{a^2 - x^2}} \tag{3}$$

Fig. 2 contains graphs of vertical stresses in the subgrade underneath a rigid foundation, estimated by Eq. (3) and FEM calculations, respectively. The stresses are defined in relation to the average stress applied onto the foundation. The stresses underneath two foundations widths are plotted. The plot displays that the discrepancy between the two methods is neglectable small.

According to Eq. (3) the stresses increase towards the edge of a foundation, a behaviour that is expected according to literature, e.g. [1]. The stresses increase along the edges due to subgrade shearing in that area. As long as an elastic material model applies, the development of shearing stresses is linearly proportional to shearing strains. The soil elements that experience shearing achieves a vertical stress component additional to the on from the volumetric compression, see Fig. 3. Hence, the soil elements outside the foundation area contribute to carrying the load. That leads to an increased subgrade response in that area. As less shearing occurs towards the centre of the foundation, the contribution of shearing in the subgrade reaction decreases towards the centre.

It should be emphasized that Eq. (3) applies to a rigid foundation only. A flexible foundation creates another shearing pattern, dependent on the load configuration. For a flexible foundation, generally, less shearing occurs at the edges and additional shearing occurs at the centre due to deflection of the foundation[1]. No foundation is fully rigid; however, the assumption of a rigid foundation is often applicable and is discussed further in section 4.1.

Furthermore, it is noted that the stress distribution applied assumes no plastic strains. Plasticity has an impact on the shearing capacity and may decrease shearing stresses in the subgrade around the foundation edge. Plasticity and its impact is discussed further in section 4.2.

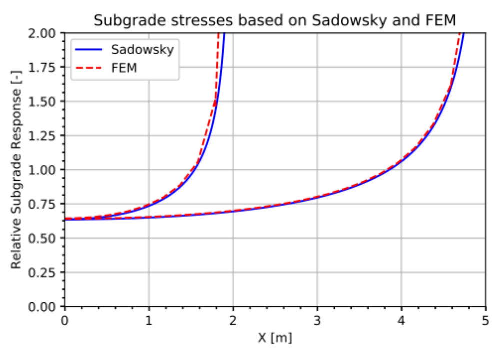

Figure 2. Vertical subgrade stresses estimated by FEM and Sadowsky's equation for two foundation widths; 4 m and 10 m.

[1] Note that dy/dx along the foundation causes shearing in the soil.

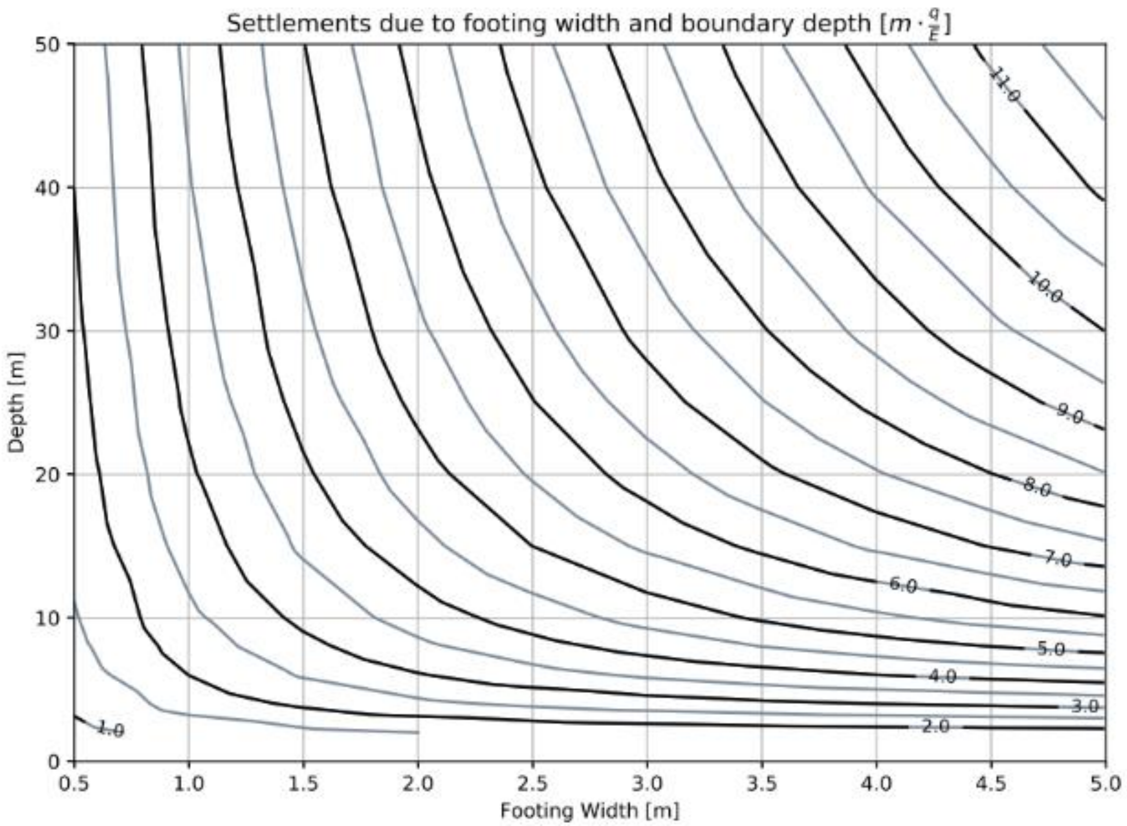

Figure 3 - Estimation of F_s

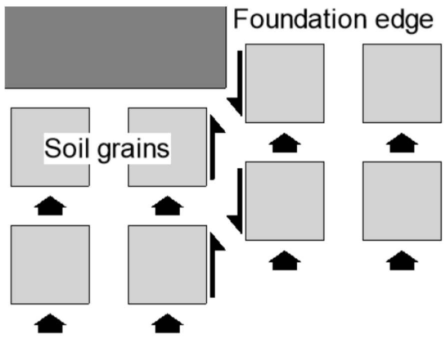

Figure 4. Shearing stress contribution along foundation edge

2.2 Settlements

A foundation settlement profile depends on several parameters. That often results in complex analyses where FEM calculations becomes handy. Considering an elastic subgrade response, the analysis simplifies significantly. For an elastically behaving subgrade, the settlements are expected to be proportional to the applied load, and inverse proportional to the E-modulus of the subgrade. On basis of that, the two remaining significant influence factors are foundation width and distance to boundary conditions.

The influence of those two factors have been evaluated and given by the contour plot in Fig. 4. The contour plot has been established on basis of 110 FEM calculations. For each calculation a rigid foundation has been modelled on top of an elastic subgrade and loaded[2]. The foundation width and distance to boundary conditions are varied between the models, and the settlement of the foundation is extracted from the models.

The factor, F_S, estimated by Fig 4 is a factor of settlements relative to the input conditions in the FEM models. The factor read from the figure can be applied to calculate settlements using Eq. (4) and to calculate the coefficient of subgrade reaction using Eq. (2).

$$S = \frac{p}{F_S} \tag{4}$$

3 BENEFITS

Several methodologies exist for calculating the coefficient of subgrade reaction. Generally, they are based on a stress distribution with a corresponding settlement profile estimated by the elastic modulus. In the following, the benefits of the method presented in this article is evaluated by comparing the corresponding subgrade stress field to one estimated by Boussinesq‘s equation for estimating vertical stresses in plane strain condition, as given by Eq. (5) [2]. The stress field considered valid for the method proposed in this article is defined by using FEM calculations.

Eq. (5) is based on assumptions like those applied in the method presented herein, however the load is applied at an infinitesimal small area. It should be noted that vertical stresses determined by Eq. (5) are very well correlated with FEM calculations[3].

$$\sigma_v = \frac{2Q}{\pi} \cdot \frac{Z^3}{\cdot(r^2+Z^2\,)^4} \tag{5}$$

The comparison is based on three calculations considering a point load applied onto a 4 m wide foundation. The foundation rigidity is varied between the calculations to observe how the subgrade stress field adapts. Foundation rigidities considered are listed below.

- Infinitely flexible – The foundation bends to an unrealistic extent, causing a point load that results in a stress propagation according to Eq. (5).
- Flexible – The foundation bends significantly, resulting in a constant an evenly distributed subgrade stress response.
- Rigid – The foundation bends very little or not at all, resulting in a stress distribution according to Eq. (3).

The comparison of stress fields is being quantified by considering the average vertical stress between a depth of 0 and 20 m.

The stresses are evaluated at the foundation centre[4] and as an average under the entire foundation[5], respectively. The results are given in Table 1 in relation to average stresses defined by Eq. (5).

According to table 1, the deviation between the subgrade stresses highly depends on the considered subgrade area. On basis of the comparison shown in Table 1, the method presented in this article may result in a stress reduction in relation to Eq. (5) between approx. 10 and 65 %.

It should be noted that the primary reduction of stresses is in the upper soil layers. Furthermore, it should be noted that an increased foundation width amplifies the reduction in relation to Eq. (5).

Table 1 - Comparison between subgrade vertical stresses

Foundation Rigidity	$\boldsymbol{\sigma_{v,x=0}}$	$\boldsymbol{\sigma_{v,x=0:2m}}$
Inf. Flexible	1.00	1.00
Flexible	0.41	0.90
Rigid	0.35	0.82

[2] Poisson’s ratio of 0.3 has been applied for the subgrade in the models.

[3] Discrepancy less than 0.1 %.

[4] x = 0.

[5] x = -2 m to 2 m.

4 DISCUSSION

In the following section, the assumptions of foundation rigidity and elastic soil behaviour is discussed. The discussion includes the impact of foundation deflection shapes. These are referred to as being happy or sad as given below.

- Happy foundation shape – A happy foundation shape constitutes a foundation centre that sinks deeper than the foundation edges.
- Sad foundation shape – A sad foundation shape constitutes foundation edges that sink deeper than the foundation centre.

It should be noted that when located onto an elastic media, an incrementally larger sinking part of a foundation achieves an incrementally increased coefficient of subgrade reaction.

4.1 Foundation Rigidity

It is evident that the behaviour of the foundation has a large impact on the subgrade stresses and settlements. Table 1 emphasize the importance of the stiffness response by the foundation and how rigidity impacts subgrade stresses and foundation settlements.

In relation to the method presented in this article it is important to define whether a foundation‘s rigidity gives rise to conservative results or not. To evaluate so, the foundation deformation shape and the externally applied loads are considered.

Foundation shapes are caused by uneven distributions of external forces (loads and subgrade reactions) impacting the foundations. Due to the uneven external forces, moments build up in a response to achieve force equilibrium. The flexural stiffness, that is a key factor concerning foundation rigidity, only impacts the degree of the foundation shape. Hence, the larger the flexural stiffness, the less pronounced the foundation shape is. Therefore, the importance of the foundation shape consideration increases with foundation flexibility.

4.1.1 Impact of a Happy Foundation Shape

For a happy foundation shape to occur, the vertical subgrade stress response at the edges has to be larger than the loads applied onto the edges. Typically, that occurs for a centrally loaded foundation, see Fig. 5.

A happy foundation shape implies increased subgrade stress response at the centre of the foundation and a lower subgrade response at the edges of the foundation. Fig. 5 illustrates the subgrade response as springs. In comparison to the rigid foundation shape, the happy foundation shape cause tightening of the springs under the centre of the foundation and relaxation of the springs under the edges of the foundation. Hence, when using Eq. (3), the vertical subgrade stresses at the centre of the foundation are underestimated and the vertical stresses at the edges are overestimated. By underestimating the vertical stress response under the loaded area, the resulting moments are underestimated. Therefore, in relation to foundation design moments, applying the methods proposed in this article is conservative for a happy foundation shape.

However, as the stresses for a happy foundation shape is centralized, the average coefficient of subgrade reaction is overestimated. The average foundation settlement is therefore expected larger than when predicted by the method proposed in this article. Furthermore, the coefficient of subgrade reaction is expected lower than when predicted by the method proposed in this article.

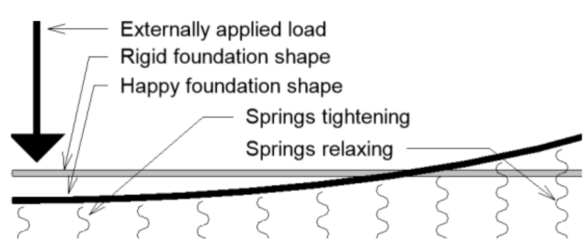

Figure 5. A happy foundation shape

4.1.2 Sad Foundation Shape

For a sad foundation shape to occur, the vertical subgrade stress response at the centre has to be larger than the loads applied onto the centre. Typically, that occurs for a foundation loaded at the edges, see Fig 6.

A sad foundation shape implies increased subgrade response at the edges of the foundation and a lower subgrade response at the centre of the foundation. Fig 6 illustrates the subgrade response as springs. In comparison to the rigid foundation shape, the sad foundation shape cause relaxation in the centre springs and tightening in the springs located at the edges. Hence, when using Eq. (3), the vertical subgrade stresses at the edges are underestimated and the vertical stresses at the centre are overestimated. By underestimating the vertical stress response under the loaded area, the resulting moments are underestimated. Therefore, in relation to foundation design moments, applying the methods proposed in this article is conservative for a sad foundation shape.

As the stresses for a sad foundation shape is drawn towards the edges, the average coefficient of subgrade reaction is underestimated. The <u>average</u> foundation settlement is therefore smaller than when predicted by the method proposed in this article. Furthermore, the coefficient of subgrade reaction is expected larger than when predicted by the method proposed in this article. Hence, for at sad foundation shape, generally, the method proposed in this article is conservative.

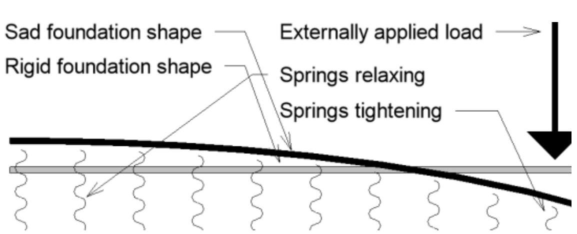

Figure 6. A sad foundation shape

4.2 Impact of Plasticity

The soil response at the edge of the foundation is higher due to propagation of vertical stresses. The propagation is made possible by shearing between soil elements. Shearing is limited by plastic behaviour that initiates if the shear stresses exceeds the capacity of the soil. The stress propagation is at its maximum for elastic materials and depending on the shear strength of the soil material it may decrease towards zero. With no possibility of shearing, the subgrade response under a rigid foundation would be evenly distributed – like water. Hence, it is evident that the shear strength and plastic behaviour have a significant impact on the subgrade response.

With poor subgrade strength characteristics, the subgrade response around the edges of the foundation may even be so low, that almost no vertical stresses can be applied without the soil initiates yielding. In this case, the foundation settles to increase the stresses at the centre, until either force equilibrium has been established or failure occurs. The area of yielding subgrade at the edge of the foundation highly depends on the soil strength characteristics and the loads size. When no yielding occurs, Sadowsky‘s equation applies to the subgrade vertical response for a rigid foundation. With yielding, the appropriate subgrade stress response may be described by either Sadowsky‘s stress distribution, evenly distributed stresses or even something in between evenly distributed stresses and a point load as defined by Boussinesq‘s equation. However, using Boussinesq‘s equation for evaluating the stresses will always be conservative. Yielding increases the total amount of settlements, and depending on how the foundation is loaded, the foundation shape adapts. In all cases, the subgrade response and the coefficient of subgrade reaction along the edges of the foundation decreases while the subgrade stress response along the centre increases. The coefficient of subgrade reaction likely, but not necessarily, decreases along the centre of the foundation, as the increased stresses are associated with additional settlements. After all, with a centrally loaded foundation, the sad foundation shape will be less pronounced with yielding, as the subgrade stress response along the edges decreases. For a foundation loaded at the edges, the sad foundation shape will be more pronounced.

It is vital to be able to estimate the degree of yielding in the subgrade‘s under a foundation when it comes to determine the subgrade response and settlements. It requires knowledge of the soil strength characteristics and the stress states in the subgrade. However, estimation of the yielding is not considered a topic of this article, and it should be emphasized that the method proposed in this article assumes that no yielding occurs.

5 EXAMPLE OF APPLICATION

1) Foundation rigidity and soil behaviour is considered for evaluation of validity of the method proposed in this article.
2) The subgrade‘s vertical stress distribution along the foundation width is determined by Eq. (3).
3) The foundation settlement is evaluated by using equation Eq. (4). The equation considers the factor Fs where the foundation width is considered in correlation to the distance to vertical boundary condition of zero displacements.
4) The coefficient of subgrade reaction may be considered along the foundation width by using Eq (2).

6 RECAP

This article has proposed a method for estimating the subgrade reactions and settlements in an elastic media when considering a rigid foundation in plane strain conditions. The method is a counterpart for calculating the stresses and settlements by using Boussinesq‘s equation. Boussinesq‘s equation applies to foundations where the loaded areas are small. This method applies to wider loaded areas in plane strain conditions.

On basis of Sadowsky‘s stress distribution and multiple FEM-calculations, the subgrade‘s vertical stresses, settlements and coefficient of subgrade reactions may be estimated. The impact of foundation rigidity and subgrade plastic behaviour should be considered when applying the method proposed in this article.

7 ACKNOWLEDGEMENT

8 SYMBOLS

p - Incremental stress $[kPa]$

S - Incremental settlements $[kPa]$

K_s - Coefficient of subgrade reaction $[kPa/m]$

M - P-wave modulus, also known as oedomoeter elastic modulus $[kPa]$

F_S - Coefficient of settlement, defined by Fig. 4 [-]

x - Considered distance from foundation center $[m]$

P - Load per meter $[kN/m]$

a - Foundation width divided by 2 $[m]$

EI - Flexural stiffness

E - Elastic modulus representative for the foundation

h - Foundation height

9 REFERENCES

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