Proceedings of the XVII ECSMGE-2019
Geotechnical Engineering foundation of the future
ISBN 978-9935-9436-1-3

doi: 10.32075/17ECSMGE-2019-0396

XVII European Conference
ECSMGE 2019
Reykjavik
on Soil Mechanics
and Geotechnical Engineering

Reliability assessment of serviceability limit state for diaphragm wall using hardening soil (HS) model

Analyse de la fiabilité de la paroi moulée réalisée dans un sol à variabilité spatiale.

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ABSTRACT: The work deals with reliability analysis of cantilever diaphragm pile wall located in spatially variable sands. The problem of serviceability limit state of the construction is considered. To precisely calculate the value of maximum wall deflection the Hardening Soil (HS) is employed as soil model. To account for soil spatial variability friction angle of the sand is modeled using anisotropic random field with specified both vertical and horizontal scale of fluctuation (SOF). The individual realizations of boundary value problem are solved using ZSoil code. The influence of horizontal SOF on the probabilistic characteristics of maximum wall deflection is tested: for assumed value of vertical SOF and few different values of horizontal SOF the probability density function of wall deflection are estimated. Based on that function the assessment of failure probability versus horizontal SOF is calculated. It is shown that in the case of diaphragm wall construction the value of horizontal SOF can significantly affect probability of serviceability failure.

RÉSUMÉ: L'article analyse le problème de la fiabilité de la paroi moulée cantilever dans des sables aux paramètres de résistance variables spatialement. Le problème des états limites en service de la paroi y a été soumis à une analyse approfondie. Afin de définir précisément la déflection maximale de la paroi, dans la déscription du sol a été adopté le modèle constitutif Hardening Soil (HS). Afin d'introduire une variabilité spatiale des paramètres pour le sol, nous avons avons présumé que la distribution spatiale de la valeur de l' angle de talus naturel du sable est définie par les champs anisotropiques aléatoires à l' échelle de fluctuation verticale et horizontale (SOF) fixée. Toutes les simulations dans le sujet analysé ont été établies à l'aide du logiciel ZSoil. Dans l'article a été analysée l'influence de la variabilité de l'échelle horizontale de fluctuation sur les probabilistes caractéristiques de la déflection maximale de la paroi. Pour la valeur fixée de l'échelle verticale de la fluctuation et plusieurs valeurs sur l'échelle horizontale de fluctuation nous avons estimé la fonction de la densité de la probabilité pour la déflection de la paroi. En se basant sur cette fonction, a été calculé la probabilité d'un état d'urgence par rapport à l'échelle horizontale de la fluctuation. Il a été démontré que dans le cas de cette construction la valeur sur l'échelle horizontale de la fluctuation peut sérieusement influer sur la probabilité d'un état d'urgence de la construction dans le contexte des états limites en service.

Keywords: random field, scale of fluctuation, friction angle, reliability index

1 INTRODUCTION

Safety concepts in geotechnics are based on an appropriate evaluation of soil properties as well as loads and then introducing them to a design process. Basic rules are given by Eurocodes in the European Union and international standard ISO 2394. However, one of the main concern of geotechnics is way of a consideration of soil properties uncertainty in the course of designing process. The complexity of the issue is related to various factors influencing this uncertainty. Among them the spatial variability is the most unpredictable and not possible to control. It is worth mentioning that the 4th edition of ISO-2394 standard contains an Annex D: Reliability of Geotechnical Structures. The annex also underlines a high impact of soil properties spatial variability on safety evaluation in geotechnics. Consequently modelling soil properties by random fields appears quite adequate. In the present study this kind of modelling is associated with finite element modelling in order to analyze displacement of diaphragm wall installed in a relatively uniform subgrade.

The aforementioned approach was already used in the past for investigating reliability of footings and other geotechnical problems (eg. Fenton and Griffiths 2003, Pieczyńska-Kozłowska et al. 2015) So far, few studies have been performed on the application of these methods to determine the reliability of supported excavations. Among them recent studies of Sert et al. (2016) and Luo et al. (2018), are the first studies (to the knowledge of the authors) which employed Hardening-Soil (HS) model for probabilistic analysis of excavation support. In both studies the effective friction angle φ' was the only random property.

The present work is complementary to this studies. It is also using HS model with the effective friction angle modelled with lognormal random field to analyze diaphragm wall installed in sands. However, the main goal of the work is to investigate how the reliability assessment of serviceability limit state is affected by value of horizontal fluctuation scale. The random field is generated using FSM algorithm. The individual realizations of the problem are solved using Z-Soil finite element (FE) code. For assumed vertical scale of fluctuation few different values of horizontal scale of fluctuation are tested.

2 MODELLING BY RANDOM FIELDS

A random field can be understood as a generalisation of a stochastic process. Given a probability space (Ω, S, P), where S is a σ-field of Ω subsets and P is a probability measure. By random field we understand a measurable function $X\colon \Omega\times R^2 \to R$ such that for each $\boldsymbol{x}=(x, y) \in R^2$, $X(\omega, \boldsymbol{x})$ is a random variable. The space R^2 means that the domain of the field is two-dimensional. A soil property that is under consideration in the domain can be spatially modelled by mapping to a given point $\boldsymbol{x_0} \in D$ the random variable $X(\omega, \boldsymbol{x_0})$. The complete probabilistic information of a certain random field is given if the family of so-called finite-dimensional distribution of the field is known (Doob 1990). The above family consists of all distributions of random vectors of the type $(X(\omega,\boldsymbol{x_1}), X(\omega,\boldsymbol{x_2}), \ldots ,X(\omega,\boldsymbol{x_m}))$, where $\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots ,\boldsymbol{x_m} \in N$ is an arbitrary taken set of points of the domain D and N is the set of natural numbers. However, in most practical cases it is difficult to identify the above family and the basic information is bounded to first two statistical moments, namely to expected value function (mean value function) $\mu(\boldsymbol{x})=E[X(\omega,\boldsymbol{x})]$ and the covariance function $C(\boldsymbol{x_1},\boldsymbol{x_2})=E\{[X(\omega,\boldsymbol{x_1})-E[X(\omega,\boldsymbol{x_1})]][X(\omega,\boldsymbol{x_2})-E[X(\omega,\boldsymbol{x_2})]]\}$, where E[.] denotes the expected value operator. It is easy to prove that a covariance function has to be a positively definite function. In application to geotechnical problems a simplification which is usually assumed is imposing the weak stationarity property that means the two following conditions must be satisfied:

- Expected value is constant along the domain under consideration (D), i.e.

$$\forall \mathbf{x} \in D \qquad \mu(\mathbf{x}) = \mathrm{E}[X(\omega, x)] = \mu \qquad (1)$$

- The covariance function is solely a function of a lag vector $\Delta\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$, i.e.

$$\forall \mathbf{x_1}, \mathbf{x_2} \in D \qquad C(\boldsymbol{v}) = \mathrm{C}(\Delta\mathbf{x}) \qquad (2)$$

Condition (2) implies that the variance is constant in the domain D (variance is equal to covariance function for $\Delta\mathbf{x}$=0), therefore a weakly stationary random fields is usually easier to handle using autocorrelation function

$$\rho(\Delta\mathbf{x}) = \frac{\mathrm{C}(\Delta\mathbf{x})}{\sigma^2} \qquad (3)$$

A correlation structure of a field defined by its covariance function plays a vital role in modelling of spatial variability of soil properties and should be selected on the base of a soil testing results. The common practice is to assume a general form of the covariance function and then determining its parameters by fitting to the test results. This implies of course, that the final results of computation depend on the selection of the covariance function form. Fortunately, in many cases, it seems that far more important is the appropriate estimation of the parameters of the covariance function than the selection of its form (see e.g. Pieczyńska-Kozłowska et al, 2017). In this study very common, usually called Markov-type (Fenton and Griffiths, 2008), correlation function was used:

$$\rho(\tau_x, \tau_y) = \exp\left(\frac{-2|\tau_x|}{\theta_x} + \frac{-2|\tau_y|}{\theta_y}\right) \qquad (4)$$

where $(\tau_x, \tau_y)=(x_2\text{-}x_1, y_2\text{-}y_1)$. Parameters θ_x and θ_y are fluctuations scales (correlation radii) in direction x and y, respectively. In one-dimensional case (direction) the scale of fluctuation can be defined as

$$\theta = 2\int_0^\infty \rho(\tau)d\tau \qquad (5)$$

It was adopted to geotechnical problems by Vanmarcke (1977,1983) and it is considered as a measure of correlation strength inside the random field. The problem of estimating the scale of fluctuation has been intensively explored in the literature (e.g. Vanmarcke 1977, Bagińska et al. 2016, Ching et al. 2017).

In some earlier papers (e.g. Griffiths and Fenton 2001, Fenton and Griffiths 2003) further simplification has been imposed by assuming the isotropy of random field. A weakly stationary random field X(ω,**x**) is called isotropic if its covariance function takes the form

$$C(\mathbf{x}, \mathbf{y}) = f(\|\mathbf{x} - \mathbf{y}\|) \qquad (6)$$

where *f*: R→R is a certain positively definite function and ‖.‖ denotes the Euclidean norm. The condition (6) means that the covariance function must be invariant with respect to any rotation of the coordinate system. However, later published papers dealing with bearing capacity of shallow foundations (e.g. Pieczyńska et al 2015) have demonstrated that the use of isotropic random fields in modelling of soil strength parameters may overestimate the bearing capacity mean value and underestimate the coefficient of variation. Due to these reasons the present study assumes anisotropy of random fields that corresponds to soil's friction angle.

The statistical sampling carried out in the present study utilizes the Monte Carlo technique. Therefore numerical simulation of random field under consideration plays the vital role. Here, basing on one of pioneering work by Fenton and Griffith (2001), the random field that model the angle of internal friction of soil is obtained from a normal random field *X* by applying the following transformation

$$Y(\mathbf{x}) = \exp(X(\mathbf{x})) \qquad (7)$$

As a result of the above transformation one-dimensional distribution of filed Y becomes the lognormal one with the probability density function (PDF) of the form

$$f(x) = \begin{cases} \frac{1}{x\,\sigma_{\ln Y}\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu_{\ln Y}}{\sigma_{\ln Y}}\right)^2\right\}, & x > 0 \\ 0, & x \le 0 \end{cases} \quad (8)$$

where parameters $\mu_{\ln Y}$ and $\sigma_{\ln Y}$ are, respectively, the mean value and the standard deviation of underlying normal distribution *X*. The mean value μ_Y and the standard deviation σ_Y of the lognormal distribution *Y* are related to $\mu_{\ln Y}$ and $\sigma_{\ln Y}$ by:

$$E[Y] = \mu_Y = \exp\left(\mu_{\ln Y} + \frac{1}{2}\sigma_{\ln Y}^2\right) \quad (9)$$

$$\sigma_Y^2 = \mu_Y^2\left(\exp\left(\sigma_{\ln Y}^2 - 1\right)\right) \quad (10)$$

3 GENERATING FE MODELS WITH RANDOM PARAMETERS

3.1 Remarks on Hardening Soil model

In order to carry out serviceability limit state analyses for several classes of soil-structure interaction problems advanced constitutive models for soils have to be used. The Hardening Soil (HS) model, formulated by Shanz (1998) and Schanz et al. (1999), and modified then by Benz (2006), who added the small strain overlay to it, is the one that proved to be a versatile tool to achieve highly accurate predictions for deformations of structures when analyzing complex problems in the domain of deep excavations, large foundation rafts, diaphragm/sheet-pile walls and many other. It is implemented in many commercial FE codes like Plaxis, ZSoil, Midas and Geo 5. In this study the ZSoil FE code (Obrzud and Truty 2010) is used to analyze individual realizations of the diaphragm wall boundary problem. The HS model belongs to the class of multisurface elastic-plastic models with generally nonassociated flow rules, deviatoric and volumetric hardening and built in the ultimate Mohr-Coulomb and Rankine strength criteria. The main advantage of this advanced model is such that most of its parameters can easily be estimated based on the laboratory triaxial drained tests, but also, based on the SDMT and SCPTU field tests as well. All the model parameters can be classified in four groups i.e. strength, dilatancy, stiffness and stress history. The group of strength parameters consists of the effective friction angle φ', effective cohesion c' (for uncemented soils this value should tend to zero) and tensile strength f_t (for uncemented soils long term tensile strength is always zero). Dilatancy phenomenon is represented by the dilatancy angle ψ. This phenomenon is strictly connected to the shear plastic mechanism and it strongly depends on the stress history. For normally consolidated cohesive soils dilatancy angle can usually be set to zero while for overconsolidated ones its value is larger than zero. In order to estimate it drained triaxial tests must be conducted for confining stress σ_3' much lower than the preconsolidation stress value. In the case of cohesionless soils dilatancy angle can be estimated as $\psi \approx \varphi'$ - 30° or $\psi \approx 1/6 \div 1/4\ \varphi'$ depending on the relative density. Stiffness in the HS model is represented by seven parameters ie. the reference Young's modulus in the regime of very small strains E_0^{ref}, the reference unloading-reloading modulus E_{ur}^{ref}, the reference secant modulus E_{50}^{ref} and tangent oedometric modulus E_{oed} defined on the primary consolidation line at a given vertical stress value, stiffness exponent *m*, equivalent shear strain $\gamma_{0.7}$ and Poisson ratio ν.

Stress dependency of the aforementioned stiffness moduli is expressed by the following power laws adopted in the standard HS model

$$E_{ur} = E_{ur}^{ref}\left(\frac{\sigma_3' + c\cot\varphi'}{\sigma_3^{ref} + c\cot\varphi'}\right)^m \quad (11)$$

$$E_{50} = E_{50}^{ref}\left(\frac{\sigma_3' + c\cot\varphi'}{\sigma_3^{ref} + c\cot\varphi'}\right)^m \quad (12)$$

$$E_0 = E_0^{ref}\left(\frac{\sigma_3' + c\cot\varphi'}{\sigma_3^{ref} + c\cot\varphi'}\right)^m \quad (13)$$

where σ^{ref} is the assumed reference stress (usually σ^{ref} =100 kPa).

For cohesionless soils the σ_3' stress dependency is not the best choice. Hence in the ZSoil code the p' can be used instead (the c cot φ' term is canceled in this version).

In the extended version of the HS model, which includes strong stiffness variation in the range of small strains, the current reference small strain stiffness modulus varies in the range $E_0^{ref} \div E_{ur}^{ref}$ depending on the strain history at a given point. This quite complicated evolution law can be found in the PhD thesis by Benz (2006). The tangent oedometric stiffness modulus E_{oed} is not the direct input parameter to the model. Its value together with the assumed value of the earth pressure coefficient K_0 for normally consolidated state $K_0^{NC} \approx 1 - \sin \varphi'$ allow to identify the two hidden model parameters that appear in the expressions for the volumetric (cap) plastic yield surface and the volumetric hardening law.

The last group of parameters concerns the stress history which must be defined to set up the size of plastic surfaces with respect to the current effective initial stress state. This definition can be made with aid of the given OCR (overconsolidation ratio) value or the overburden pressure q^{pop}. It has to be emphasized that the second definition yields a strongly nonlinear OCR(z) profile observed in the CPT or DMT field tests. In the considered case a cohesionless case is considered therefore the OCR=1 is assumed. The comprehensive report concerning this model and its calibration can be found in the work by Obrzud and Truty (2010).

3.2 FE analysis of diaphragm wall with random field of strength properties

In the analyzed problem the 10m long diaphragm wall protecting the 5m deep excavation was assumed to be installed in the spat~~e~~ially variable layer of sand. The FE model of the analyzed problem is shown in Fig 1. The subsoil was discretized with aid of EAS-Q4 elements, which are locking free for incompressible and strongly dilatant geomaterials, while the diaphragm wall was discretized with aid of the two node Timoshenko beam elements. Contact frictional interface elements was added between the diaphragm wall and adjacent subsoil. Width of the full model was 50m, depth - 20m, width of the excavation was 15m and excavation depth was 5m. The excavation process was realized in five stages. The assumed uniform grid size used to discretize subsoil was 0.2m. This yields 25000 elements in the subsoil domain and 20 beam elements in the wall. Diaphragm wall was assumed to be 0.6m thick, with Young Modulus E_D=30000 MPa.

In the present study the effective friction angle φ' was the only parameter assumed to be random with parameters of lognormal distribution μ_φ=34° and σ_φ=3.4° (coefficient of variation (COV) of φ' was equal 10%). The underlying normal field with parameters obtained using equation (9) and (10) was generated using Fourier Series Method (FSM) (Jha and Ching 2013). The correlation function was assumed of the form (4). The values of vertical and horizontal SOF used in the analysis are discussed in the next section.

In the context of the HS model the value of φ' changing from element to element influences not only the ultimate state soil behavior but also intermediate states. For the sake of consistency it was assumed that the resulting initial K_0 state, dilatancy angle and friction angle, in the interface, φ_i are also ~~random~~ changing locally with their values depending on the local value of friction angle φ'_{loc} in the following manner

$$K_{0\,loc} = 1 - \sin \varphi'_{loc} \tag{14}$$

$$\tan \psi_{loc} = \frac{\tan \varphi'_{loc}}{\tan \varphi'_{glob}} \tan \psi_{glob} \tag{15}$$

$$\tan\ \varphi_{i\,loc} = \frac{\tan \varphi'_{loc}}{\tan \varphi'_{glob}} \tan \varphi_{i\,glob} \tag{16}$$

The initial values of soil parameters are as follows: E_0^{ref} =225.6MPa, $\gamma_{0.7}$=0.00015, R_f=0.9, OCR=1, E_{ur}^{ref} =90 MPa, ν=0.2 σ^{ref} =100 kPa, m=0.54 E_{50}^{ref} =30 MPa, E_{oed} =30 MPa at vertical stress 227 kPa, φ'_{glob}=34°, ψ_{glob}=6°, c=0.2 kPa,.

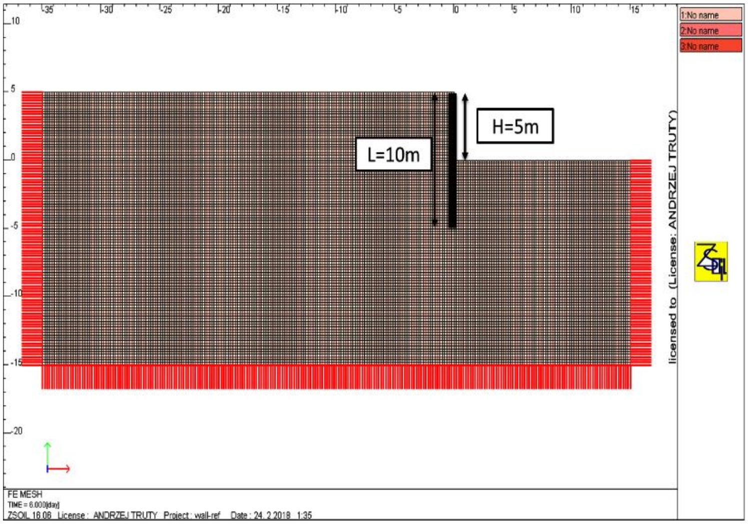

Figure 1 FE discretization

For the purpose of probabilistic modeling the Monte Carlo simulations were carried out (5000 runs for each assumed stochastic distribution model). The whole process was automatized with aid of the Python scripts that allowed us to modify the input (*.dat) file for ZSoil calculation module and then analyze the results. To perform further reliability analyses the maximum deflection of the wall were extracted using ZSoil Python SDK.

4 RESULTS OF SIMULATIONS

The main objective of preformed simulations was to determine the influence of horizontal SOF value on the probability distribution of diaphragm pile wall maximum deflection. For this purpose for assumed value of vertical SOF equal 0.5m four different values of horizontal SOF, namely 2m, 5m, 10m and infinity were tested. As was mentioned before for each of the assumed values 5000 MCS were performed. Based on the collected data in each case, the probability distribution of the results was estimated and tested with the goodness of fit Kolmogorov test. For the estimated parameters of lognormal distribution, the test results were in the range of 22-95% (which for 5000 realization seem to be quite satisfactory results). There was no reason to reject the hypothesis that the obtained results follow the estimated distributions. The estimated distributions of deflection obtained for different values of horizontal SOF have been presented in Fig 2. As can be seen the impact of horizontal fluctuation scale on the results is significant.

This impact can be even better observed on the diagrams of mean values and coefficients of variation (COV) of maximum wall deflection presented in Figs 3 and 4, respectively. The obtained values of mean maximum deflection are almost identical for differently assumed horizontal SOF, however some trend for garter values obtained for higher SOF is visible. This trend is confirmed by some other, not presented results. Much more clear trend can be observed on the diagram of COV. The value of deflection COV is changing from 0.065 obtained for horizontal SOF equal 2m to 0.1 for horizontal SOF equal infinity.

It is obvious that the change in probability characteristics influences also the reliability assessment of serviceability limit state. The convenient measure of reliability is the reliability index β associated with failure probability as $p_f=\Phi(-\beta)$ where Φ is the distribution function of the standard normal distribution. In the case under consideration the probability of failure p_f is understood as an exceeding by wall deflection certain allowable value. For the assumed allowable wall deflection equal to 0.025 m, obtained indexes β versus horizontal SOF are presented in Fig 5. As can be seen the indexes for differently assumed horizontal SOF are changing from 1.0 to 1.8 which corresponds to the change in failure probability from 3% to 14% (the index decreases with horizontal SOF, and failure probability inversely - its maximum value occurs for horizontal SOF equal to infinity). Apparently the horizontal SOF can significantly influence reliability of diaphragm wall.

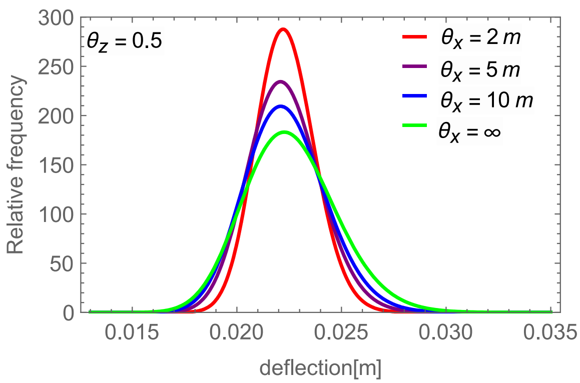

Figure 2 Deflection distributions

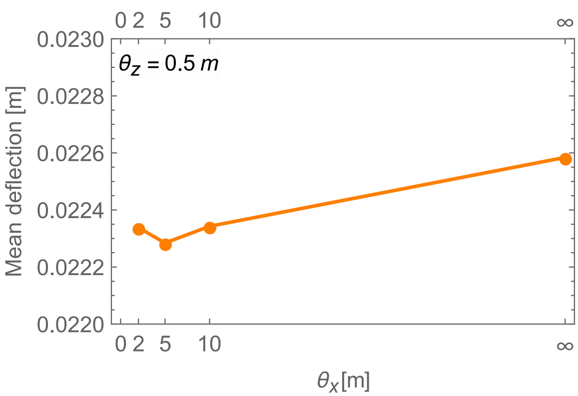

Figure 3 Mean values of the deflection

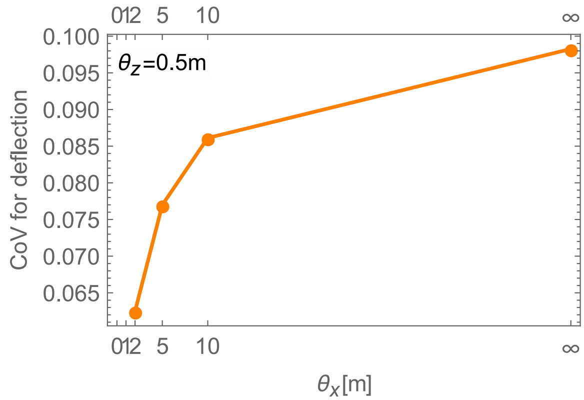

Figure 4 COV of wall maximum deflection

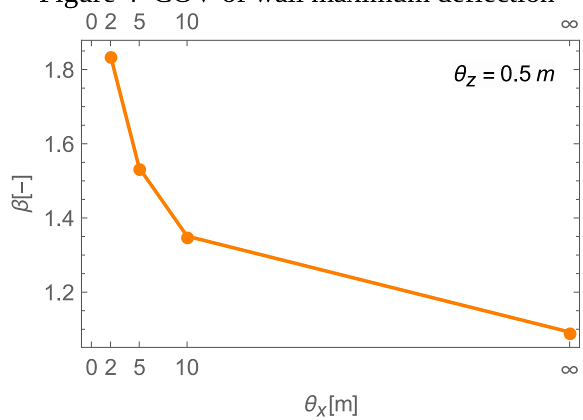

Figure 5 Reliability index β versus horizontal SOF

5 CONCLUSIONS

In this paper serviceability limit state reliability assessment of diaphragm wall was investigated. The advanced constitutive model, namely HS model was assumed for the soil. The main objective was to estimate the impact of horizontal SOF on the wall reliability. It was shown that horizontal SOF can significantly influence the value of probability of serviceability limit state failure.

Probabilistic modeling allows estimating the reliability of a structure. However, since numerical modeling is used, the precision of that estimation depends on precision of numerical calculation. To take into account serviceability limit state and more accurately determine the values of internal forces for excavation supporting structures, one should reach for more advanced calculation models. The use of models such as HS with random parameters seems to be the right path.

In previous works using the HS model for probability analysis of supporting structures (Sert et al 2016, Luo et al 2018) a horizontal SOF was assumed as infinity. Although from the presented results it seems that this is the most conservative approach, it may be uneconomical. In order to design structures more economically, a horizontal SOF should be examined. In the view of recent research (e.g. Ching et al 2018), it seems that it will soon be possible to estimate value of horizontal SOF even using the data from limited testing points.

6 REFERENCES

Bagińska, I., Kawa, M., Janecki, W. (2016). Estimation of spatial variability of lignite mine dumping ground soil properties using CPTu results. *Studia Geotechnica et Mechanica*, *38*(1), 3-13.

Benz T., 2006, *Small-strain stiffness of soils and its numerical consequences*, PhD thesis, University of Stuttgart

Ching, J., Wu, T. J., Stuedlein, A. W., Bong, T. 2017. Estimating horizontal scale of fluctuation with limited CPT soundings. *Geoscience Frontiers*.

Doob, J. L. *Stochastic processes*. 1990. New York, John Wiley & Sons.

Fenton, G.A, and Griffiths, D.V., 2003. Bearing capacity prediction of spatially random c - φ soils, *Canadian Geotechnical Journal*, 40(1), 54—65.

Fenton G. A., Griffiths, V. D. 2008. *Risk assessment in geotechnical engineering*. John Wiley & Sons.

Griffiths, D. V., Fenton, G. A. 2001. Bearing capacity of spatially random soil: the undrained clay Prandtl problem revisited. *Geotechnique*, *51*(4), 351-360.ISO 2394:2015 *General principles on reliability for structures*. International Organization for Standardization ISO

Jha, S. K., Ching, J., 2013, Simplified reliability method for spatially variable undrained engineered slopes. *Soils and Foundations*, *53*(5), 708-719.

Lunne, T., Robertson, P. K., Powell, J. J. M., 1997. *Cone penetration testing in Geotechnical Practice*. Taylor & Francis Ltd.

Luo, Z., Li, Y., Zhou, S., Di, H., 2018. Effects of vertical spatial variability on supported excavations in sands considering multiple geotechnical and structural failure modes. Computers and Geotechnics, 95, 16-29.

Obrzud R., Truty A., *The hardening soil model. A practical Guidebook*, Technical report ZSoil.PC 100701, Lausanne, 2010.

Pieczyńska-Kozłowska, J. M., Puła, W., Griffiths, D. V., Fenton, G. A. (2015). Influence of embedment, self-weight and anisotropy on bearing capacity reliability using the random finite element method. *Computers and Geotechnics*, *67*, 229-238.

Pieczyńska-Kozłowska J.M., Puła W., Vessia G., 2017 A collection of fluctuation scale values and autocorrelation functions of fine deposits in Emilia Romagna Palin, Italy. *Geo-Risk 2017*; 290-299 Reston: American Society of Civil Engineers

Schanz T., *Zur Modellierung des mechanischen Verhaltens von Reinbungsmaterialien*, Mitt. Inst. für Geotechnik 45, Universitat Stuttgart, 1998.

Schanz T., Vermeer P.A., Bonier P.G., *Formulation and verification of the Hardening Soil Model*, Beyond 2000 in Computational Geotechnics, Rotterdam, 1999.

Sert, S., Luo, Z., Xiao, J., Gong, W., & Juang, C. H., 2016. Probabilistic analysis of responses of cantilever wall-supported excavations in sands considering vertical spatial variability. *Computers and Geotechnics*, 75, 182-191

Vanmarcke E.H., 1977. Probabilistic Modeling of Soil Profiles. *Journal of the Geotechnical Engineering Division*, 103(GT11), 1227-1246.

Vanmarcke E.H., 1983. *Random Fields – Analysis and Synthesis*, Cambridge, MIT Press