Proceedings of the XVII ECSMGE-2019
Geotechnical Engineering foundation of the future
ISBN 978-9935-9436-1-3

doi: 10.32075/17ECSMGE-2019-0388

Prediction of pile settlement using simplified models

Évaluation du tassement des pieux par des modèles simplifiés

J. J. Crispin, P. J. Vardanega, G. Mylonakis
University of Bristol, Bristol, UK

ABSTRACT: Modelling soil nonlinearity in pile settlement analysis often requires numerical analysis based either on 1-dimensional "t-z" curves or even multi-dimensional constitutive models, which may not be appealing to geotechnical engineers. A number of alternative approaches are available that take advantage of simplified constitutive models and analytical methods to develop solutions that are suitable for spreadsheet calculations. This paper investigates the error introduced by the assumptions in two such models. A theoretical "t-z" curve is developed based on a simplified constitutive model and a new approximate approach developed that incorporates nonlinearity for both shaft and base resistance. This is compared to the predictions of a linear elastic – perfectly plastic method using an informed choice of linear elastic stiffness. A basic sensitivity study is carried out comparing the two approaches with a 1-dimensional numerical solution.

RÉSUMÉ: La modélisation de la non linéarité d'un sol vis à vis du tassement d'un pieu nécessite une analyse numérique basée sur des courbes unidimensionnelles «t-z» ou sur des modèles de comportements multidimensionnels, ce qui n'attire peu les ingénieurs géotechniques. D'autres approches existent, permettant de simplifier le modèle et les méthodes analytiques pour développer des solutions appropriées à un tableur. Cet article montre l'erreur induite par les hypothèses de ces deux modèles. Une courbe théorique «t-z» a été développée sur un modèle simplifié et une nouvelle approche approximative, qui intègre le comportement non linéaire de la résistance au frottement latéral du pieu et de sa base. Ceci est comparable aux prédictions d'une méthode élastique linéaire - parfaitement plastique utilisant un choix judicieux de la rigidité élastique linéaire. Une étude basique de sensibilité est menée en comparant les deux approches avec une solution numérique à une dimension.

Keywords: piles & pilling; settlement; soil/structure interaction.

1 INTRODUCTION

Rigorous analysis of single piles under axial load requires considering the full continuum problem. Analytical solutions are available based on a linear elastic soil model (e.g. Mylonakis 2001a, 2001b, Anoyatis et al. 2019). Numerical analysis can be employed to model nonlinear soil behaviour but such approaches are often time consuming.

The problem can be reduced to 1 dimension through the introduction of empirical or analytical "t-z" curves (e.g. Coyle and Reese 1966). The pile can then be discretised and an iterative numerical method employed. This is the basis of many commonly used design software such as Oasys Pile (Oasys 2017) and Ensoft TZPile (Reese et al. 2014).

As an alternative, analytical solutions have been developed that are suitable for hand or spreadsheet calculation. These allow for rapid estimates of foundation performance at early stages of design. In this paper two such approaches and the simplifying assumptions they employ are compared.

2 MODIFIED LOAD-TRANSFER APPROACH

Vardanega et al. (2012) developed a simple analytical method based on a nonlinear soil model. The soil undrained shear stress-strain response is described using the following power-law function (Vardanega and Bolton 2011):

$$\frac{1}{M} = \frac{\tau}{c_u} = \frac{1}{2}\left(\frac{\gamma}{\gamma_{M=2}}\right)^b, 1.25 \leq M \leq 5 \tag{1}$$

where M is a mobilisation factor on undrained shear strength, c_u, τ is the applied shear stress, $\gamma_{M=2}$ is the shear strain, γ, at $0.5c_u$ and b is a positive exponent describing soil nonlinearity. Equation (1) was calibrated by Vardanega and Bolton (2011) using a database of mechanical soil tests on a variety of clays and silts, the average parameter values were $b=0.6$ and $\gamma_{M=2}=8.8\times10^{-3}$.

The concentric cylinder model used by Cooke (1974) and Randolph and Wroth (1978) yields equation (2) which describes how the shear stress, τ, decreases with distance from the pile, r.

$$\tau = \frac{\tau_0 D_s}{2r} \tag{2}$$

where τ_0 is the shear stress on the pile shaft and D_s is the pile shaft diameter.

By substituting the undrained shear stress-strain relationship in equation (1) into equation (2) and integrating the shear strain with respect to the distance from the pile, Vardanega et al. (2012) derived a form of equation (3), which was generalised in Vardanega (2015), describing the displacement of the pile, w, due to an applied shear stress at the pile circumference, τ_0:

$$w = \frac{b\cdot\gamma_{M=2}\cdot D_s}{2(1-b)}\left(\frac{2\tau_0}{c_u}\right)^{\frac{1}{b}} \tag{3}$$

Equation (3) is valid until slip occurs at the pile-soil interface. Skempton (1959) defined an empirical adhesion factor, α, on undrained shear strength, c_u, such that slip occurs when $\tau_0 = \alpha\, c_u$.

Rearranging equation (3) and multiplying the shear stress by the circumference of the pile allows the relationship to be expressed as a "t-z" curve:

$$t = \begin{cases} t_u\left(\frac{w}{w_y}\right)^b & w < w_y \\ t_u & w \geq w_y \end{cases} \tag{4}$$

where t is the applied skin friction per unit length (units of force per length), t_u is the ultimate skin friction per unit length, given by equation (5), w_y is the displacement at which the "t-z" curve transitions to plastic behaviour, given by equation (6) and b is the same soil nonlinearity exponent from equation (1).

$$t_u = \alpha c_u \pi D_s \tag{5}$$

$$w_y = \frac{b\cdot\gamma_{M=2}\cdot D_s}{2(1-b)}(2\alpha)^{\frac{1}{b}} \tag{6}$$

There is no known closed form solution available to calculate the head settlement, w_0, of a compressible pile using a nonlinear "t-z" curve. Instead, equation (7) was developed (Vardanega et al. 2012, Vardanega 2015), the first term of which gives the pile head settlement due to soil deformation (shearing), assuming a rigid pile. The second term is the compression of the pile itself under the applied load, which is assumed to act at the midpoint of the pile.

$$\frac{w_0}{D_s} = \frac{b\cdot\gamma_{M=2}}{2(1-b)}\left(\frac{2}{M}\right)^{\frac{1}{b}} + \frac{2}{M}\frac{\overline{c_u}}{E_p}\left(\frac{L}{D_s}\right)^2 \tag{7}$$

where M in this case is the average mobilisation of shear strength along the pile shaft, $\overline{c_u}$ is the average undrained shear strength over the length of the pile, L, and E_p is the elastic modulus of the pile.

Following Vardanega et al. (2018) and conservatively assuming that the base resistance is not mobilised until full exhaustion of the shaft resistance, the mobilisation factor, M, can be related to the applied pile head load, P, using equation (8):

$$\frac{P}{P_u} = \frac{1}{M}\left[\alpha + \frac{N_c}{4}\frac{c_{ub}}{\overline{c_u}}\frac{D_s}{L}\left(\frac{D_b}{D_s}\right)^2\right]^{-1} \quad (8)$$

where P_u is the ultimate load the pile can support, calculated by summing the ultimate shaft and base resistance of the pile, P_{us} and P_{ub}, respectively, given by (Skempton 1959):

$$P_u = P_{us} + P_{ub} = \alpha\overline{c_u}\pi D_s L + N_c c_{ub}\frac{\pi D_b}{4} \quad (9)$$

where N_c is the undrained bearing capacity factor (Skempton 1951 conservatively recommends a value of 9 for deep foundations), c_{ub} = the soil undrained shear strength under the pile base and D_b = the pile base diameter.

This method has performed well at predicting the response of piles embedded in London Clay at different sites (Vardanega et al. 2018, Voyagaki et al. 2018), however, its main limitation is that it does not incorporate a model for the base response. This means that any base resistance mobilised before the shaft resistance is exhausted is neglected, and predictions are limited to $P < P_{us}$.

2.1 New approach

In this paper a model following similar principles is developed that also incorporates base response. Skempton (1951) remarked that the load-settlement curve for a pad foundation is similar in shape to the stress-strain curve of an undrained triaxial test on a representative soil sample. This indicates that the pile base response can be modelled using the following "P_b-w_b" curve:

$$P_b = \begin{cases} P_{ub}\left(\frac{w_b}{w_{b,y}}\right)^b & w_b < w_{b,y} \\ P_{ub} & w_b \geq w_{b,y} \end{cases} \quad (10)$$

where w_b is the settlement of the pile base when the load applied is P_b, $w_{b,y}$ is the pile base settlement when the base resistance is first exhausted ($P_b = P_{ub}$) and b is the soil nonlinearity exponent from equation (1).

In a similar model, Williamson et al. (2017) assumed that the pile base response is fully mobilised once the base settlement, w_b, reaches 10% of the base diameter, D_b. This assumption for the failure criterion of the pile base is often attributed to Terzaghi. However, this has been shown to disagree with Terzaghi's original recommendations. Fellenius (2013) states in this regard: "Most certainly, Terzaghi did not suggest that a fixed movement value, however determined, could serve as a definition of capacity."

As an alternative, Skempton (1951) and Osman and Bolton (2005) have proposed factors linking the strain in a representative soil sample to the displacement of a pad foundation at a certain mobilisation. Skempton (1951) hypothesises that this factor is independent of the depth of a foundation. However, the authors are not aware of any work to validate this hypothesis.

Combining the base and shaft resistance for a elastic pile is a non-linear problem and as closed form solutions are not available, simplifying approximations must again be applied. Assuming the pile is rigid, the settlement along the shaft and at the base are equivalent (denoted here as w_r). The total load carried by the pile at this settlement equals the sum of the shaft and base load:

$$P = P_{us}\left(\frac{w_r}{w_y}\right)^b + P_{ub}\left(\frac{w_r}{w_{b,y}}\right)^b \quad (11)$$

Equation (11) can be rearranged to give the settlement due to the soil deformation, w_r, at a given load, P:

$$w_r = \left[\frac{P}{P_{us} \cdot w_y{}^{-b} + P_{ub} \cdot w_{b,y}{}^{-b}}\right]^{\frac{1}{b}} \quad (12)$$

The compression of the pile, Δw, can then be calculated by assuming the load on the pile acts at a certain depth, L_c:

$$\Delta w = \frac{P \cdot L_c}{E_p A_p} \tag{13}$$

where A_p = the pile cross sectional area. Setting L_c as half the pile length would be equivalent to the assumption in Vardanega et al. (2012). Fleming (1992) recommended a value of $0.45L$ when considering a similar problem.

The total head settlement, w_0, is therefore given by summing equations (12) and (13). Equations (11)-(13) are valid until the shaft resistance is exhausted. This occurs when $t=t_u$ in equation (4), therefore the settlement required assuming a rigid pile is w_y, given in equation (6). Inputting this into equation (11) gives the load at which this change in behaviour occurs, P_i:

$$P_i = P_{us} + P_{ub}\left(\frac{w_y}{w_{b,y}}\right)^b \tag{14}$$

Note that P_i is larger than, P_{us}, as a portion of the base resistance is mobilised before full exhaustion of the shaft resistance.

The settlement of the pile after this load is reached can then be calculated by summing the rigid pile settlement and elastic shortening as before. Any additional load will be taken only by the pile base, acting over the entire length, L, of the pile, therefore w_0 is given by:

$$w_0 = w_{b,y}\left(\frac{P-P_{us}}{P_{ub}}\right)^{\frac{1}{b}} + \frac{P_{us} \cdot L_c}{E_p A_p} + \frac{(P-P_{us}) \cdot L}{E_p A_p} \tag{15}$$

The full pile load-settlement curve can be generated by first calculating P_i with equation (14), then summing equations (12) and (13) to give w_0 for a range of head load values between 0 and P_i, and employing equation (15) to give w_0 for a range of head load values between P_i and P_u.

At large loads, when the base resistance is nearly fully mobilised, the valid range of equation (1) may be exceeded. Skempton (1951) also found there was less agreement with his experimental data for high values of P_b/P_{ub}, however, most piles are not designed to carry loads in this range.

3 MODIFIED WINKLER APPROACH

Closed form analytical solutions are available if the pile is instead assumed to be supported by linear elastic – perfectly plastic “t-z” curves (e.g. Scott 1981, Guo 2012, Crispin et al. 2018). In the linear elastic range, these methods can even be extended to calculate pile-pile interaction factors for pile group analysis in closed form (Mylonakis and Gazetas 1998, Crispin and Leahy 2019).

By assuming that the ultimate skin friction per unit length, t_u, is reached first at the pile head, and propagates down the pile, the pile can be split into two sections. These are the “plastic” section in the upper part of the pile, of length L_p, where $t=t_u$, and below this the “elastic” section, of length $L_e=L-L_p$, where $t = kw$ and k is the gradient of the “t-z” curve, known as the Winkler spring stiffness or modulus of subgrade reaction (Scott 1981).

Considering the plastic section first and assuming t_u is constant with depth, P and w_0 may be determined by treating the pile as an elastic rod with known base load and distributed shaft load:

$$P = P_{Lp} + t_u L_P \tag{16}$$

$$w_0 = w_{Lp} + \frac{P_{Lp}}{E_p A_p} L_P + \frac{t_u}{2E_p A_p} {L_P}^2 \tag{17}$$

where P_{Lp} and w_{Lp} = the load and settlement at the interface depth, L_p, respectively. w_{Lp} is by definition the displacement at which the “t-z” curve transitions to plastic behaviour, w_y:

$$w_y = t_u / k \tag{18}$$

P_{Lp} can be calculated by multiplying w_{Lp} by the stiffness of the elastic section, K_{el}. In homogeneous soil (k is constant with depth), this is given by (Mylonakis and Gazetas 1998):

$$K_{el} = E_p A_p \lambda \frac{\Omega + tanh(\lambda L_e)}{1 + \Omega tanh(\lambda L_e)} \tag{19}$$

where λ is a load transfer parameter and Ω is a dimensionless base stiffness constant:

$$\lambda = \sqrt{\frac{k}{E_p A_p}}\ ,\ \Omega = \frac{K_b}{E_p A_p \lambda} \tag{20}$$

where K_b is the gradient of the "P_b-w_b" curve.

The full load-settlement curve can be generated by calculating the head load and settlement for L_p values in the range $0 \leq L_p < L$, then employing equations (16) and (17) with $L_p = L$, $P(L_p) = P_b$ and $w(L_p) = w_b$ for a range of w_b values until the base resistance is exhausted.

3.1 Selecting k and K_b

Traditionally the Winkler spring stiffness, k, is calculated by matching the 1D solution to a numerical or analytical continuum solution (e.g. Randolph and Wroth 1978, Mylonakis 2001a). However, the nonlinear "t-z" curve in equations (4-6) allows an informed choice of this stiffness to be made.

The energy, W, required to increase t from 0 to t_u is given by:

$$W = \int_0^{t_u} w \cdot dt \tag{21}$$

where w = the pile displacement at that depth. By calculating this energy for both the "t-z" curve in equation (4) and a linear elastic – perfectly plastic "t-z" curve with stiffness k and equating them, the result can be rearranged to give an equivalent linear stiffness (shown in Figure 1):

$$\frac{k}{t_u} = \frac{1-b^2}{b^2 \cdot \gamma_{M=2} \cdot D_s} (2\alpha)^{\frac{-1}{b}} \tag{22}$$

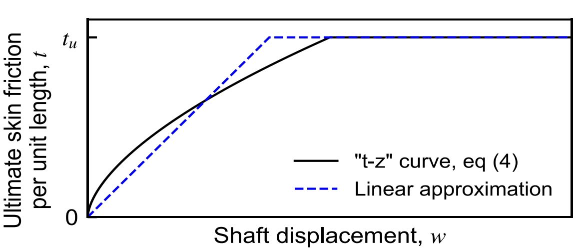

Figure 1. "t-z" curve and linear approximation

A similar process can be employed to select a base spring stiffness, K_b. However, in this case the response only has to remain linear until the shaft resistance is exhausted. This occurs when the base displacement, w_b, is equal to the yield displacement of the "t-z" curve at that depth, w_y. Therefore, the energy stored in the "P_b-w_b" curve when $w_b = w_y$ can be equated to the energy stored by a base spring at this same displacement and the result rearranged to give the spring stiffness:

$$\frac{K_b}{P_{ub}} = \frac{2}{w_y} \int_0^{w_y} \frac{P_b}{P_{ub}} dw_b = \frac{2}{(1+b)} \frac{{w_y}^{b-1}}{{w_{b,y}}^{b}} \tag{23}$$

This can then be connected to the base load and settlement at failure to give a bilinear "P_b-w_b" curve, shown in Figure 2.

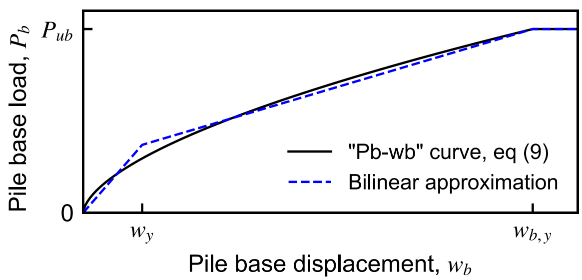

Figure 2. "P_b-w_b" curve and bilinear approximation

3.2 t_u and k varying with depth

Closed-form analytical solutions for a limited set of k and t_u variations with depth are also available in the literature (Scott 1981, Guo 2012, Crispin et al. 2018). For a linear variation of t_u with depth, equations (16) and (17) can be replaced by equations (24) and (25), respectively (Crispin et al. 2018):

$$P = P_{Lp} + \frac{t_{u,Lp} + t_{u0}}{2} L_P \tag{24}$$

$$w_0 = w_{Lp} + \frac{P_{Lp}}{E_p A_p} L_P + \frac{2t_{u,Lp} + t_{u0}}{6 E_p A_p} {L_P}^2 \tag{25}$$

where t_{u0} and $t_{u,Lp}$ = the ultimate skin friction per unit length at the pile head and the interface depth, L_p, respectively.

For a linear variation of k with depth, equation (19) can be replaced by equation (26) (Crispin et al. 2018):

$$K_{el} = E_p A_p \lambda_L \frac{S_1+\Omega_L S_2}{S_3+\Omega_L S_4}\sqrt{\frac{k_{Lp}}{k_L}} \tag{26}$$

where k_L and k_{Lp} are the values of k at the pile base and L_p, respectively, λ_L and Ω_L are inhomogeneous parameters analogous to λ and Ω:

$$\lambda_L = \sqrt{\frac{k_L}{E_p A_p}}\ ,\ \Omega_L = \frac{K_b}{E_p A_p \lambda_L} \tag{27}$$

and S_1, S_2, S_3 and S_4 are given by:

$$\begin{aligned} S_1 &= I_{-2/3}(\chi_0)I_{2/3}(\chi_L) - I_{2/3}(\chi_0)I_{-2/3}(\chi_L) \\ S_2 &= I_{-2/3}(\chi_0)I_{-1/3}(\chi_L) - I_{2/3}(\chi_0)I_{1/3}(\chi_L) \\ S_3 &= I_{-1/3}(\chi_0)I_{-2/3}(\chi_L) - I_{1/3}(\chi_0)I_{2/3}(\chi_L) \\ S_4 &= I_{-1/3}(\chi_0)I_{1/3}(\chi_L) - I_{1/3}(\chi_0)I_{-1/3}(\chi_L) \end{aligned} \tag{28}$$

where $I_v(\ )$ is the modified Bessel function of the first kind, of order v, and χ_0 and χ_L are given by:

$$\chi_0 = \frac{2\,k_{Lp}\,\lambda_L L_e}{3(k_L - k_{Lp})}\sqrt{\frac{k_{Lp}}{k_L}}\ ,\chi_L = \frac{2\,k_L\,\lambda_L L_e}{3(k_L - k_{Lp})} \tag{29}$$

4 COMPARISON

In order to compare the two approaches, a basic sensitivity study has been conducted. The problem set up is shown in Figure 3, a range of parameters are considered to test the performance of both solutions for a large number of practical configurations.

A one-dimensional (1D) numerical solution has been developed using the mumerical Runge-Kutta method recommended by Williamson et al (2017) for comparison with the two approaches. The predicted response for this method and the two approximate approaches are shown in Figure 4, axes scaled to the region of most interest.

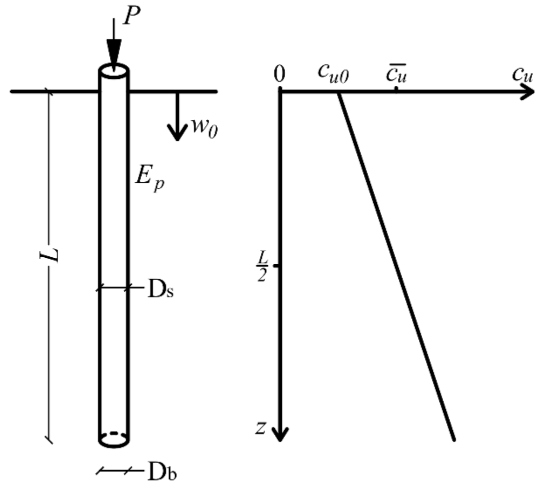

Figure 3. Problem dimensions

Figure 4(a) shows the results for a straight-shafted (D_b=D_s) example pile with a slenderness ratio, $L/D_s = 20$. The soil undrained shear strength increases linearly to a base value three times the value at the surface, c_{u0}, therefore $c_{u0}/\bar{c}_u = 0.5$. A representative pile soil stiffness ratio of $E_p/G_{s,M=2}=3000$ has been chosen, where $G_{s,M=2}$ is the secant shear modulus when half of the undrained shear strength has been mobilised. For a concrete pile of $E_p \approx 20GPa$ and a soil with $\gamma_{M=2}=8.8\times10^{-3}$ (the average value in the Vardanaga and Bolton 2011 database) this represents a soil with $\bar{c_u} \approx 120kPa$. Finally, $b=0.6$ was chosen (the average value in the Vardanega and Bolton 2011 database).

The remaining plots show the effect of varying different parameters. Varying base diameter is shown in Figure 4(b), pile slenderness in Figures 4(c) and 4(d), soil inhomogeniety in Figures 4(e) and 4(f), pile-soil stiffness ratio in Figures 4(g) and 4(h) and soil nonlinearity exponent in Figures 4(i) and 4(j). Both approaches perform well compared to the numerical results. However, they both overestimate settlement for slender piles and piles with a low stiffness, shown in Figures 4(d) and 4(g) respectively.

5 SUMMARY AND CONCLUSIONS

A new approximate approach has been developed to account for nonlinearity of both the pile shaft and base when predicting settlement. Additionally, a simple process to select a Winkler spring stiffness, k, when modelling a nonlinear "t-z" curve has been demonstrated. Both approaches matched a 1D numerical analysis well for a variety of configurations. However, care should be taken when modelling soft, slender piles.

6 ACKNOWLEDGEMENTS

The authors acknowledge the support of "Databases to INterogate Geotechnical Observations (DINGO) EP/P020933/1 EPSRC. The first author is supported by EPSRC, grant number EP/N509619/1. Thanks are due to P. Moireaud for his assistance preparing the French abstact. The work reported in this paper has not generated new experimental data.

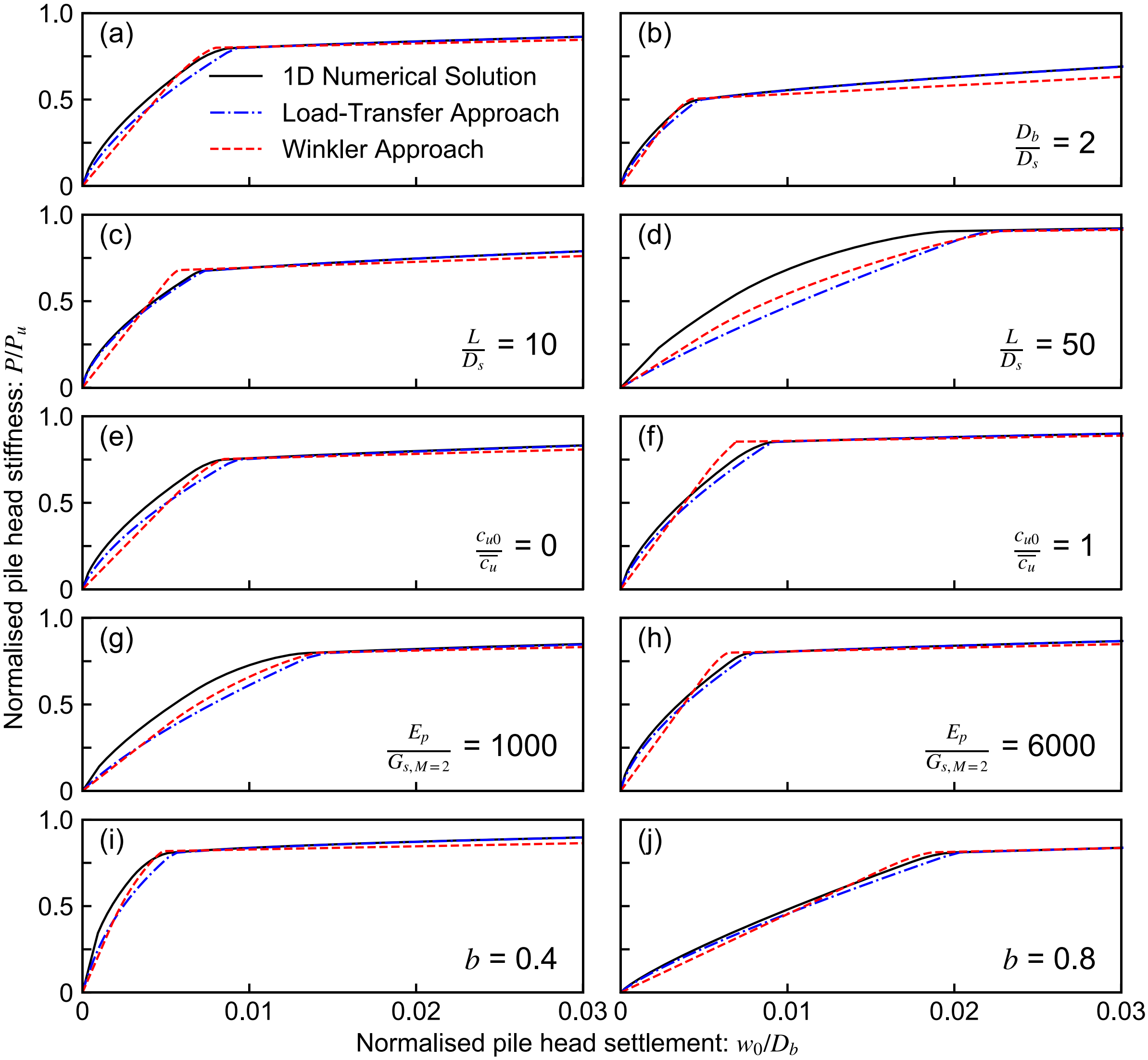

Figure 4. Comparison of the predicted load-settlement curves for each method. Unless otherwise noted, the following parameters have been used: $D_b/D_s = 1$, $L/D_s = 20$, $c_{u0}/\bar{c_u} = 0.5$, $E_p/G_{s,M=2} = 3000$, $b = 0.6$.

7 REFERENCES

Anoyatis, G., Mylonakis, G., Tsikas, A. 2019. An analytical continuum model for Winkler modulus of axially-loaded end-bearing piles in inhomogeneous soil. *International Journal for Numerical & Analytical Methods in Geomechanics*, 10.1002/nag.2886.

Cooke, R.W. 1974. The settlement of friction pile foundations. *Building Research Station Current Paper*, CP 12/75.

Coyle, H.M., Reese, L.C. 1966. Load transfer for axially loaded piles in clay. *Journal of Soil Mechanics and Foundations Division* **92(2)**, 1-26.

Crispin, J.J., Leahy, C.P., Mylonakis, G. 2018. Winkler modulus for axially-loaded piles in inhomogeneous soils. *Géotechnique Letters* **8(4)**, 290-297.

Crispin, J.J., Leahy, C.P. 2019. Settlement of axially loaded pile groups in inhomogeneous soil. *Journal of the Deep Foundation Institute*, 10.1080/19375247.2019.1588535.

Fellenius, B.H. 2013. Discussion of "Load tests on full-scale bored pile groups". *Canadian Geotechnical Journal* **50(4)**, 451-453.

Fleming, W.K.G. 1992. A new method for single pile settlement prediction and analysis. *Géotechnique* **42(3)**, 411-425.

Guo, W.D. 2012. *Theory and practice of pile foundations*. CRC Press. Boca Raton, FL, USA.

Mylonakis, G. 2001a. Winkler modulus for axially loaded piles. *Géotechnique* **51(5)**, 455-461.

Mylonakis, G. 2001b. Elastodynamic model for large-diameter end-bearing shafts. *Soils and Foundations* **51(5)**, 455-461.

Mylonakis, G., Gazetas, G. 1998. Settlement and additional internal forces of grouped piles in layered soil. *Géotechnique* **48(1)**, 55-72.

Oasys. 2017. Pile Oasys geo suite fore Windows. Available at: www.oasys-software.com/wp-content/uploads/2017/11/pile19.6_manual.pdf , [accessed 22nd March 2019].

Osman, A.S., Bolton, M.D. 2005. Simple plasticity-based prediction of the undraided settlement of shallow circular foundations on clay. *Géotechnique* **55(6)**, 435-447.

Randolph, M.F., Wroth, C.P. 1978. Analysis of deformation of vertically loaded piles. *Journal of Geotechnical Engineering* **104(12)**, 1465-1488.

Reese, L.C., Wang, S.T., Arrelaga, J.A. 2014. TZPILE 2014 user's manual. ENSOFT INC. Austin, TX, USA. Available at: www.ensoftinc.com/products/tzpile/doc/tzpile-users-manual.pdf, [accessed 22nd March 2019].

Scott, R.F. 1981. *Foundation Analysis.* Prentice-Hall. Eaglewood Cliffs, NJ, USA.

Skempton, A.W. 1951. Bearing capacity of clays. In *Building Research Congress*, London, UK, **1**, 180-189.

Skempton, A.W. 1959. Cast in-situ bored piles in London Clay. *Géotechnique* **9(4)**, 153-173.

Williamson, M.G., et al. 2017. Open-face tunnelling effects on non-displacement piles in clay – part 1: centrifuge modelling techniques. *Géotechnique* **67(11)**, 983-1000.

Vardanega, P. J. (2015). Sensitivity of simplified pile settlement calculations to parameter variation in stiff clay. In *Proc XVI ECSMGE*. ICE Publishing, London, UK, **7**, 3777-3782.

Vardanega, P.J., Bolton, M.D. 2011. Strength mobilization in clays and silts. *Canadian Goetechnical Journal* **48(10)**, 1485-1503; corrigendum **49(5)**, 631.

Vardanega, P.J., Williamson, M.G., Bolton, M.D. 2012. Bored pile design in stiff clay II: mechanisms and uncertainty. *Proceedings of the ICE – Geotechnical Engineering* **165(4)**, 233-246; corrigendum **166(5)**, 518.

Vardanega, P.J., et al. 2018. Geodatabases to improve geotechnical design and modelling. *ce/papers* **2(2-3)**, 401-406.

Voyagaki, E., et al. 2018. Analytical approaches to predict pile settlement in London Clay. In *Sustainability Issues for the Deep Foundations (eds El-Naggar et al)*, 162–180.