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Destructuring / restructuring model for quasi-static behavior of granular soils – concept of target structure

Modèle de déstructuration/ restructuration pour le comportement quasi-statique des sols – concept de la structure cible

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ABSTRACT: In this paper, one of the main assumptions related to soil fabric and its evolution with the stress state, considered in an elasto-plastic constitutive model previously proposed by the authors, is re-visited based on additional assessment of literature results. The new evidence justifies the introduction of a target soil structure concept towards which the current soil fabric continuously progresses, while the target structure instantaneously evolves with the stress state. The relationship between the target structure and the current stress tensors can be described by a power law of which coefficient value is suggested by the literature data.

RÉSUMÉ: Dans ce papier on discute une des principales hypothèses concernant la structure des sols et son évolution avec l'état de contrainte, présentée précédemment dans une loi élasto-plastique proposée par les auteurs. L'hypothèse est discutée sur la base des analyses de données présentées dans la littérature. Ces observations justifient l'introduction d'un concept de structure cible, vers lequel la structure actuelle évolue progressivement, alors que la structure cible évolue instantanément avec l'état de contrainte. La relation entre le tenseur de structure cible et le tenseur de contrainte peut être décrite par une loi de puissance dont la valeur de l'exposant est suggérée par des données de la littérature.

Keywords: fabric/structure of soils, constitutive relation, fabric – stress state relationship

1 INTRODUCTION

Soils are complex materials made by particles undergoing complex interactions. Two main approaches can be used to approach their behaviour. The first is a macroscopic approach based on continuum mechanics and described by the plasticity theory (Salençon 1974). Eventually, a framework allowing to define yielding and failure of anisotropic soils was established originated on the theory of representations for tensor functions (Boehler, & Sawczuk, 1977). The second is based on microscopic approaches considering the spatial arrangement of particles and soil voids through a fabric characterization of the soil regarded as an assembly of discrete particles (Brewer, 1964).

The continuum approach generates constitutive relations (Wood, 1991) that can be implemented in realistic numerical analysis of boundary value problems. However, the way in which the applied stresses are supported by the so-called force chains is controlled by the discrete nature of the particulate system. It is then appropriate to study the soil fabric and integrate it into a continuum constitutive model (Wan & Guo, 2004). Several forms of micro-macro constitutive laws unifying these two approaches were therefore proposed beginning with the early 1980's (Oda & Nakayama 1989). However, rather surprisingly, the construction of an integrated theory for granular soils is still in progress. Some fundamental aspects like, for instance, the usefulness of the construction of a critical state framework considering the fabric anisotropy is still questioned (Dafalias 2016). This undeniably reflects the difficulties and challenges to construct a simple and reliable generalized theory to include a granular structure/fabric.

The present paper discusses one of the main assumptions of an elasto-plastic constitutive model proposed by the authors in a previous work (Cazacliu & Ibraim 2016) associated to the soil fabric tensor and its evolution. After a brief presentation of the model, discussion is focused on the concept of a fabric target tensor as an intermediate parameter to define the relationship between the current structure of the material and the current stress state. Under loading, the target structure instantaneously evolves with the stress state, while the current soil structure continuously progresses, in time, towards the target structure. In the process, the induced fabric anisotropy is implicitly considered. Model details and simulation results are presented in Cazacliu & Ibraim (2016) work.

2 PROPOSED MODEL

The proposed continuum constitutive model follows an elasto-plastic modelling framework and is formulated in a structure space defined as an isomorphic transformation (Mac Lane, 1971) of the stress space. Inspired by thixotropic-type frameworks (Coussot, 2007), the model is considering a structure parameter (of tensorial form) the evolution of which illustrates the competition between two soil fabric mechanisms: the continuous restructuration towards a stable configuration and its destructuration under shear. In this context, it is postulated that the tensor structure's evolution rate depends on its current value and on the current stress rate.

The structure space is defined as a 3-dimensional space with the axes representing the three principal directions of a material structure tensor and the formulation of the second-order structure tensor, N_{ij}, used by Radjai et al. (2012), which associates the coordination number and the structure tensor as defined by Satake (1982) is adopted:

$$N_{ij} = n \int_{\Omega} \xi_i \, \xi_j E(\xi) d\xi \tag{1}$$

where ξ_i is the inter-particle contact normal in the i-direction, $E(\xi)$ is the contact normal distribution function (spatial probability density function of ξ), Ω is the space of contact orientations. $n = tr\left(N_{ij}\right)$ designates the coordination number, half of the average number of contacts per particle. N_{ij} eigenvalues could be considered as representative indicators for structure intensity over the principal directions (Maeda and Ibraim, 2008).

A target stress state dependent structure tensor, $N_{ij\infty}$ is introduced and it is assumed that the current material structure tensor, N_{ij}, tends in time towards the target structure:

$$\dot{N}_{ij} = \lambda\left(N_{ij\infty} - N_{ij}\right) = \lambda \cdot \Gamma_{ij} \tag{2}$$

where λ is a material constant and $\Gamma_{ij}(N_{ij})$ is the structure parameter tensor defined by the distance between the target and the current material structure. If the stress state changes, the target structure, $N_{ij\infty}$, instantaneously changes and the structure parameter, Γ_{ij}, generally increases; this describes the destructuration mechanism. At sufficiently low stress rates, the structure parameter tensor decreases, and relation (2) characterises the restructuring mechanism. It is further assumed that the target structure tensor is proportional with the current stress state tensor following a power law:

$$N_{ij\infty} = \frac{n_\infty}{tr({\sigma_{ij}}^m)} {\sigma_{ij}}^m \tag{3}$$

where $n_\infty = tr(N_{ij\infty})$ is the target coordination number, tr(●) indicating the trace of the tensor ● and *m* is a material constant which controls the evolution of the distortional part of the target structure, $N_{ij\infty}$, with the distortional part of the stress σ_{ij}.

Proportionality between: $(n_\infty - n_{cr})$, the difference between the target coordination number and the coordination number at the critical state, n_{cr}, and $(\eta_\infty \pm \eta_{max})$, the distance between a hardening variable associated to the target structure state, η_∞, and a failure limit in the direction of loading, $\pm\eta_{max}$ yields the following incremental form relation:

$$\dot{n}_\infty = \frac{n_\infty - n_{cr}}{\eta_\infty - sig(\dot{\eta}_\infty)\ \eta_{max_\infty}} \dot{} \tag{4}$$

which suggests that the target coordination number, n_∞, converges monotonically towards the coordination number at the critical state, n_{cr}, once η_∞ changes. In this approach, the failure limit η_{max} is assumed state independent, and confounded with the value of the hardening variable at the critical state.

Details of the elastic-plastic framework, including the forms of the yield surface, $f_y(N_{ij}, \eta)$, flow rule, plastic potential function, $g(N_{ij}) = \Gamma_{ij}\Gamma_{ij}$, and hardening rule, are given in Cazacliu and Ibraim (2016). Figure 1 gives a graphic description of the yield and plastic potential functions in the meridian structure plane, including the structure, target structure and their incremental evolutions $\delta N_{ij\infty}$and δN_{ij}.

Figure 2 presents the flow chart of the model in stress-controlled mode, including the final deduced relations for the elastic, ${\dot{\varepsilon}_{ij}}^e$, and plastic, ${\dot{\varepsilon}_{ij}}^p$, incremental strain rate tensors. C_{ijkl} is the elastic compliance fourth-order tensor and $\dot{\sigma}_{kl}$ is the stress rate tensor, $\dot{\eta}$ is the rate of the hardening parameter, a is a material constant which scales the plastic distortional strain, and $n_q = \sqrt{\frac{2}{3}J_2(N_{ij})}$ where $J_2(N_{ij})$ is the second invariant of structure tensor deviatoric part.

The model uses 7 parameters: 2 for the elastic (E_{max} and ν) and 2 for the plastic (η_{max} and a) behaviours, 2 for the target structure (m and n_{cr}) and one for the restructuration mechanism (λ). Example of simulation of soil behaviour under various loading conditions are presented in Cazacliu & Ibraim (2016) and they emphasise the requirement for a value of about 0.33 of the *m* parameter. Discussion on the concept of fabric tensor as an intermediate parameter to define the relationship between the current structure of the material and the current stress state is presented in the following sections.

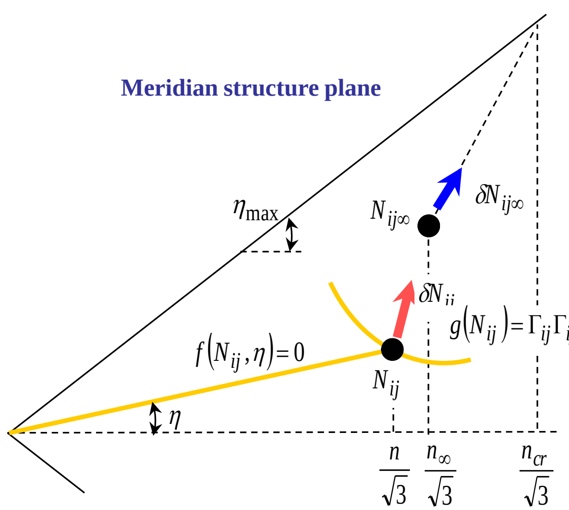

Figure 1. Schematic presentation of the yield and potential functions in the meridian structure plane, including the structure, target structure and their incremental evolution.

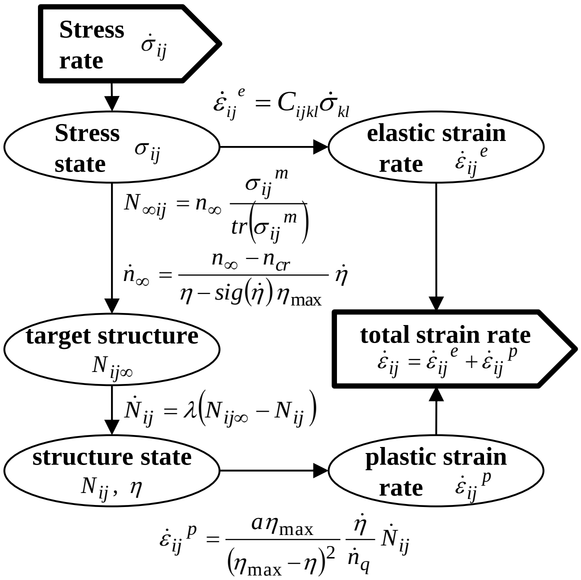

Figure 2. Flow chart of the model.

3 FABRIC EVOLUTION UNDER LOADING

3.1 Influence of the stress state

One key desirable ingredient for a generalized approach including the soil fabric is the relation between the contact anisotropy and the stress tensor. These two are recognized as closely related by several studies spanning over several decades. For instance, Oda, Konishi and Nemat-Nasser (Oda et al. 1980) observed that the principal fabric axes tend toward those of the stress during load paths implying continuous or discontinuous rotation of principal stress axes. They concluded that the contact fabric gradually changes in a manner to allow the material to withstand the increased applied stresses, including gain of contact over directions closely parallel to the major compressive stress. The existence of a quantitative relation between the contact fabric tensor and the stress state was suggested, especially in the post-failure behavior, reflecting also an important loss of contacts in those directions parallel to the minor compressive stress (Tobita 1989). However, a reliable constitutive relationship between the stress and the contact fabric tensors is proving hard to emerge.

Kuhn et al. (2015) evaluated three forms of contact anisotropy tensors, defined by the average contact orientations, the branch vectors and the mixed contact – branch orientations respectively, with their variations, averaging only the strong contacts, in line with the work of Radjai et al. (1998). They compare the evolutions of these six measures of contact orientations (differences between the maximum and minimum principal values weighted by the trace of the tensors) with the normalized deviator stress generated by DEM simulations. In all cases, the tensors using all contacts are poorly correlated to the stress and only the strong contact orientation tensor correlates well at low stresses. The closest relation was obtained for the strong mixed contact – branch tensor as presented in the Figure 3. However, even in this case, the deviator stress evolved much faster than the fabric deviator. The tangent proportionality between the deviators is far from being constant during the loading, and increases from about 0.65 at the beginning of the loading to about 0.80 at the critical state.

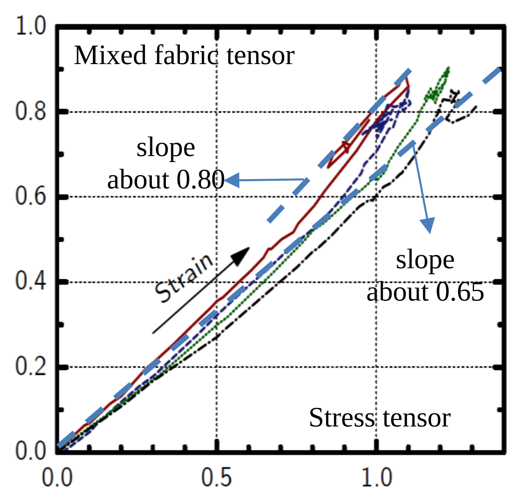

Figure 3. Correspondence between stress and mixed fabric tensors deviators during biaxial compression, for four particles shapes (red line for spherical particles) during strains of 0-60% (data from Khun et al. 2015)

Kruyt (2012) investigated the relationship between the stress tensor σ_{ij} and dimensional contact fabric tensor F_{ij} of the form:

$$\sigma_{ij} = a_O F_{ij} + a_1 F_{ik} F_{kj} \quad (5)$$

He concluded that no functional relation between the principal values of the stress tensor and of the fabric tensor could be observed. However, a relationship between the ratio of the principal values of the fabric tensor and the ratio of the principal values of the stress tensor up to the peak shear strength may be defined (Figure 4).

More recently, Shi and Gao (2018) also analyzed the fabric of the whole contact network in the pre- and post-peak deformation stages by DEM simulations. They concluded that the fabric could be approximatively related to the applied stresses through a proportionality between the principal stress values and the principal fabric values at a power of 0.4. That was also previously suggested by Oda (1982) with values of the power index between 0.4 and 0.6. Chowdhury and Nakai (1998) suggested a power index of 0.5.

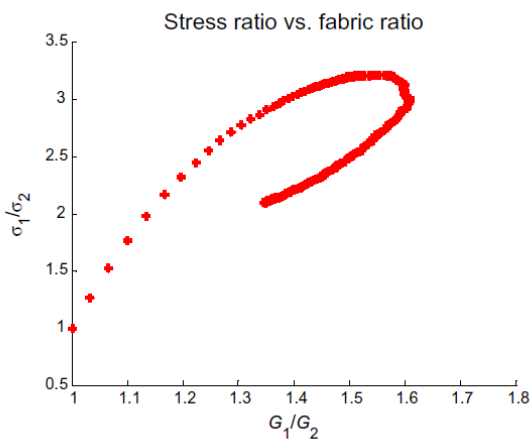

Figure 4. Fabric ratio (here G_1/G_2) vs. Stress ratio (from Kruyt 2012)

3.2 Relationship at very large strains

When analyzing more in detail the results presented by Shi & Gao (2018), one can observe that the relation between the stress and fabric tensors could be considered only at large strains, higher than 10 or 15%. In addition, the estimation of the power index was not very accurate.

In order to explore further the relationship between the fabric and the stress tensors at very large stains, we re-visit here the results previously presented by Kruyt (2012) and Kruyt & Rothenburg (2016). It is remembered that the isobaric bi-axial deviator tests were approached by two-dimensional DEM simulations on disk-shaped particles. The contact fabric was determined based on the contact normal vectors. Figure 5 presents the evolution of the maximum to minimum contact fabric ratio function of the maximum to minimum stress ratio. In this present case, the two tensors are coaxial and the ratio quantities are expressed function of principal values. The numerical results shown in Kruyt & Rothenburg (2016) were conducted up to a very large strain level of 40%. The paths corresponding to the dense packing and the loose packing were different but the final point in the Figure 5 appears to be the same. This final point corresponds to the critical state.

The numerical simulations by Kruyt (2012) were stopped at a much smaller final strain, 20%. It seems probably that the critical state hasn't been yet reached at this intermediate strain level. However, the tendency of both dense and very dense packing seems to converge towards the same critical state point.

The critical state point is located on the curve corresponding to a power law relationship between the fabric and stress ratios, with a power index of 0.35. In all cases, at the beginning of the loading from an isotropic stress and fabric, so in small strain domain, the stress ratio evolved faster than the fabric (power index of about 0.25). This suggests that the fabric cannot change fast enough when the strain rate level is low. Indeed, for the loose packing, which develops higher strain rates than the denser material, at similar stress ratios, the power index increased rather faster and converged directly to the value corresponding to the critical state. For the dense packings, a peak value of stress was observed. It is to be noted that the peak value corresponded to about 2% in strain. After the peak value, the stress ratio began to decrease, but, at the first stages, the fabric ratio continued to increase. Once the critical state is approaching, the fabric ratio also decreases, and its decrease rate is faster than that of the stress ratio, as observed at the critical state. Indeed, the stress evolves slowly once the material is approaching the critical state, it seems that the fabric evolves more significantly.

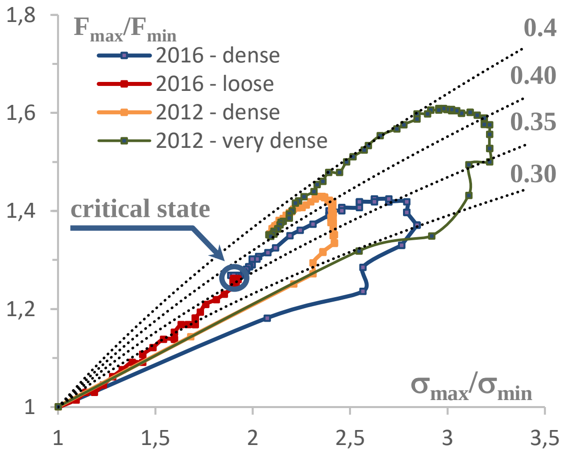

Figure 5. Fabric ratio vs. Stress ratio; the curves for power law dependency of the fabric ratio with the stress ratio are given for four power indexes 0.30, 0.35, 0.40 and 0.45 (initial data from Kruyt 2012 - marked 2012 in the figure - and Kruyt & Rothenburg 2016 - marked 2016 in the figure – re-analyzed in the present work)

A similar analysis is made in the Figure 6. The data is obtained for the work of Gu et al. (2017). Drained compression and extension triaxial tests were simulated in three-dimensional DEM on spherical particles. The fabric tensor was deduced by dividing the Young’s modulus normalized by the stress in the corresponding direction, with an index power of 1/3 (corresponding to Hertz-type contacts). The strain levels are not provided, but it is apparent that the numerical tests were stopped at a low strain value, probably less than 2%.

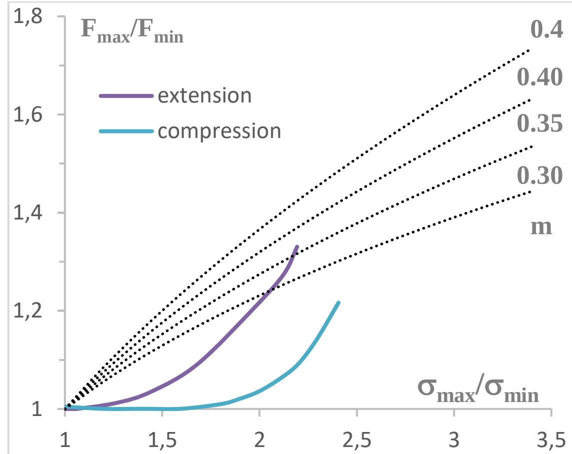

Figure 6. Fabric ratio vs. Stress ratio (initial data from Gu et al. 2012)

Nonetheless, these results confirm the tendency observed on two-dimensional numerical simulations at low levels of strain. In extension, smaller strain level is needed to reach the critical state. One can consider that the final point of the fabric – stress ratio evolutions, in extension, is not so far from the critical state. The corresponding power index is again, about 0.35.

Based on these results, one can conclude that the trends observed for the fabric and stress ratios relationship, are of similar nature for extension and compression, two- and three-dimensional particle systems, as well as for loose and dense packings.

3.3 The “Anisotropic critical state theory”

A fabric tensor and its evolution toward a critical value, norm-wise and direction-wise was postulated by Li & Dafalias (2012). The critical fabric tensor was a function of the loading direction, mainly the direction of plastic flow. It is suggested that for simple situations, the direction can be related to the stress tensor for monotonic radial loading. When the direction of loading changes, the critical fabric also changes and the fabric evolves towards the new target.

4 TARGET STRUCTURE

Certainly, the coaxiality between the strain rate tensor and the stress tensor is not always observed in practice (Drescher & de Josselin de Jong 1972). The coaxiality of these tensors with the fabric tensor is neither verified too. From this simple observation, it is then clear that an assumption of a proportionality between the fabric tensor and the stress tensor, eventually with a power index, could not be reasonably postulated. The results presented in the Figures 5 and 6 demonstrate that even if the coaxiality is guaranteed, the assumption of proportionality is not valid.

The conjecture of a target fabric at the critical state (Li & Dafalias 2012) to which the current fabric continuously evolves during the loading seems a convenient assumption. When the level of the strain rate is low, the fabric evolves slower and is delayed comparing with the stress evolution. At high strain rates, as when approaching the critical state, the fabric overtakes the delay.

In our model we generalize this critical state target structure, by considering that a different target structure exists at each level of stress, and the relation between the stress tensor and the fabric tensor is the same as the relationship at the critical state. At any stress state, a target fabric is defined by one-to-one correspondence and the current structure evolves towards this target structure. Under loading, as the target structure evolves to the target structure characterizing the critical state, the current structure itself evolves to the critical state target structure.

Li & Dafalias (2012) also suggest that the evolution towards the critical state target structure is produced under the loading. In our model we also generalize this assumption, by considering that the evolution is depending on the time too. Indeed, in practice it was clearly observed that when the evolution of the stress state (the loading) is stopped, the packing continues to develop strain. In our model, a fix stress state corresponds to a fix target structure, but the current structure continues to evolve towards the target structure. When the stress state is changing again, the target structure is instantaneously changed, and the current fabric continues to evolve towards the new target.

5 CONCLUSIONS

The anisotropy tensor and the stress tensor are recognized as closely related by several studies spanning over several decades. However, no functional relation between the principal values of the stress tensor and of the fabric tensor could be observed. The only emerging relationship is a power law between the fabric and stress ratios, observed mainly at very large strain. It also seems to be confirmed that under the loading the current structure tensor principal directions continuously evolve towards the stress principal directions.

Based on these observations, the model presented by Cazacliu & Ibraim (2016) proposes a way to overcome these limitations by introducing an intermediate fabric parameter which is the target structure. In this way, the current structure does not depend directly on the stress tensor but evolves continuously to this target structure. It is the target structure which is defined by one-to-one correspondence with the stress state. Simulation results presented in the previous author’s work confirm the validity of the approach.

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