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KIM – Numerical studies on the spherical cavity expansion problem

KIM – Études numériques du problème d'expansion de cavité sphérique

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ABSTRACT: The paper describes the challenges that arise during execution and quality control of deep vibro-compaction works on land reclamation projects. The achieved relative density can be measured indirectly by performing Cone Penetration Tests (CPT). Well established empirical correlations between the cone resistance q_c and the relative density I_D are not suitable to reproduce the characteristics of carbonate sands, as commonly used for land reclamation projects, because these sands show significantly lower q_c-values than silica sands under similar conditions. An alternative to traditional correlation methods to derive the q_c-requirement is the use of the Karlsruhe Interpretation Method (KIM). This paper deals with the numerical simulation of the spherical cavity expansion problem as part of the KIM. After a short introduction to the theory of this interpretation method a curve-fitting procedure is proposed which can be used for the determination of the KIM-parameters required for the approximation of the spherical cavity expansion limit pressures. A detailed description of the FE-model to solve the spherical cavity expansion problem is presented. Further, a full KIM-analysis of a land reclamation project in Dubai using a hypoplastic constitutive model is discussed. Additionally, a sensitivity analysis is performed and a comparison of the proposed FE-model with a finite-difference code is presented.

RÉSUMÉ: L'article décrit les défis qui se posent lors de l'exécution et du control de qualité de travaux de vibrocompactage profond, menés dans le cadre de projets d'amélioration des sols. La densité relative obtenue peut être mesurée indirectement en utilisant des Essais de Pénétration au Cône (CPT). Les corrélations empiriques établies entre la résistance du cône q_c et la densité relative I_D, couramment utilisés lors des projets d'amélioration des sols, ne sont pas appropriés pour reproduire les caractéristiques des sables carbonatés, puisque ce type de sable présente des valeurs de q_c nettement plus faibles que le sable siliceux dans des conditions similaires. L'utilisation de la méthode « Karlsruhe Interpretation Method » (KIM) est une alternative aux méthodes de corrélation traditionnelles pour obtenir les paramètres requis. Cet article présente des simulations numériques du problème d'expansion de cavité sphérique dans le cadre de KIM. Une procédure d'ajustement de courbe, qui peut être utilisée pour la détermination des paramètres requis pour l'approximation des pressions limites d'expansion de cavité est proposée. Une description du modèle numérique utilisé pour résoudre le problème d'expansion de cavité est également présentée. De plus, une analyse du type KIM d'un projet d'amélioration de sols, performé à Dubaï en utilisant un modèle constitutif hypoplastique, est discutée. Après une analyse de sensibilité, une comparaison du modèle en éléments finis avec un code des différences finis est présentée.

Keywords: spherical cavity expansion; numerical modelling; hypoplasticity; model sensitivity

1 INTRODUCTION

The quality control of compaction measures such as vibro compaction is related to a certain accomplished level of density. In order to make densification requirements tangible for contractors, they are converted into technical parameters such as the relative density I_D which can be directly measured using the sand replacement or the cutter cylinder method. A way of indirectly determining the density state of a compacted soil mass is to perform CPT tests and use appropriate correlations between the measured cone penetration resistance q_c and the relative density I_D.

2 KARLSRUHE INTERPRETATION METHOD

One interpretation method that could efficiently be used for the estimation of the relative density in calcareous sands is the semi-emperical Karlsruhe Interpretation Method (KIM) by Cudmani (2000). The key point of this method involves the numerical solution of a spherical cavity expansion (SCE) problem using a hypoplastic constitutive model. Thereby, a spherical cavity gets expanded inside a hypoplastic continuum from an inital radius a_0 until a stationary expansion pressure p_{LS} is reached. The limit pressures p_{LS} are then further on related to the penetration resistance q_c by the introduction of a so-called shape factor k_q.

$$q_c = k_q(I_D) \cdot p_{LS}(p_{0,}I_D) \tag{1}$$

where the initial relative density I_D and the initial effective mean pressure p_0 represent the material state. The hypoplastic equation can be calibrated to the site-specific material with conventional lab tests. Meier (2007) showed that in case of a proper calibration of the hypoplastic model, also the effect of grain crushing, which is a very typical behaviour of calcareous sands containing high amounts of breakable shells, can be incorporated by this method.

2.1 Approximation of the limit pressures

According to Cudmani (2000) the limit pressures p_{LS} obtained from the spherical cavity expansion can be approximated by the following equation which is a function of the material state:

$$p_{LS} = a \cdot {p_0}^b \tag{2}$$

Where the factor a and exponent b are

$$a = a_1 + \frac{a_2}{a_3 + I_D} \tag{3}$$

$$b = b_1 + \frac{b_2}{b_3 + I_D} \tag{4}$$

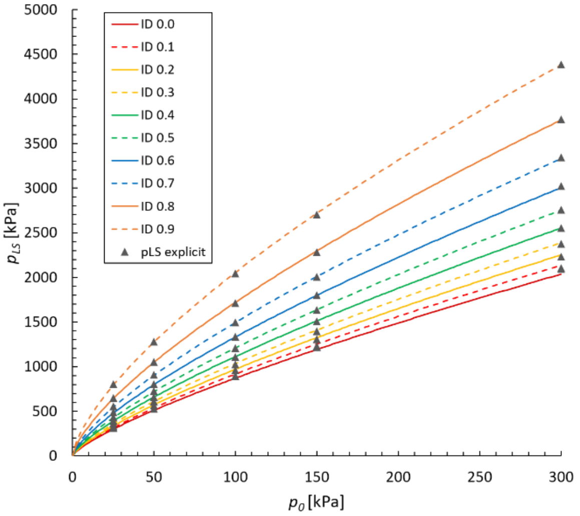


Figure 1. Example: Approximation of p_{LS}-values

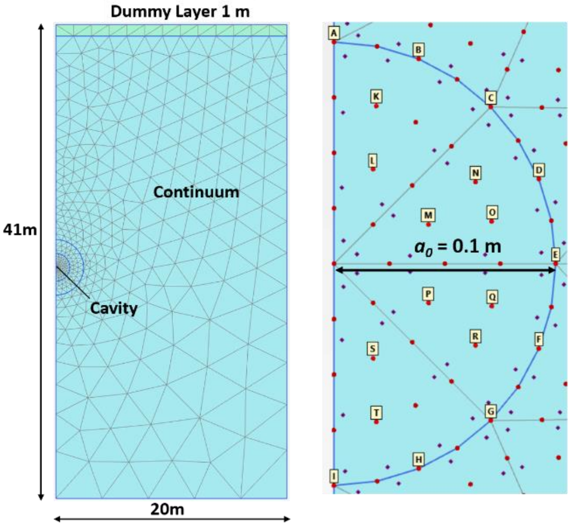
The a_i- and b_i - parameters from eqns. (3) and (4) are known as the KIM-parameters and can be found for a specific material by curve-fitting of explicit values for p_{LS} recieved from a series of SCE-simulations performed at different inital relative densities and effective mean pressures. Figure 1 shows the p_{LS}-curves from the approximation of the explicit p_{LS} -values obtained from 50 SCE-simulations performed at 5 different initial effective mean pressures p_0 and 10 different relative densities I_D.

2.2 Curve-fitting procedure

The first part of the least square curve-fitting procedure involves the approximation of the explicit p_{LS} values calculated at one relative density I_D using eqn. (2). By minimizing the sum of the squared errors ($= \Sigma[p_{LS,explicit} - p_{LS,approx}]^2$) between the explicit p_{LS}-values and the approximated values at the corresponding mean pressures p_0, the a- and b-values related to each single I_D can be found. Having these values determined the next step is to find the a_i- and b_i-parameters. The final values for a_i can be found by the minimization of the sum of the squared errors ($= \Sigma[a_{,explicit} - a_{,approx}]^2$) between the previously determined explicit a-values at different relative densities I_D and the approximated a-values (eqn. (3)) at the corresponding relative densities. In the same way it is possible to come up with the b_i-parameters when using eqn. (4). Once the KIM-parameters for the investigated material are known, the values for the cone penetration resistance q_c can be calculated from eqns. (1) – (4) for each desired values of the effective mean pressure p_0 and the relative density I_D.

3 FINITE ELEMENT MODEL OF THE SPHERICAL CAVITY EXPANSION

Cudmani and Osinov (2001) solved the spherical cavity expansion problem to determine the limit pressures p_{LS}. The solution was implemented into a finite difference code developed by Osinov, refered to as the OSINOV code, which uses the hypoplastic model acc. to von Wolffersdorff (1996). To allow for the opportunity to use different soil models, the spherical cavity expansion problem was modelled by means of the finite element software PLAXIS 2D (Bringkreve et al. 2017) in the present studies.



(a) (b)

Figure 2. (a) Typical mesh and (b) selected output nodes acc. to Xu (2007)

3.1 Modelling in PLAXIS 2D

The model setup of the spherical cavity expansion in PLAXIS is largely based on the studies by Xu (2007). The model properties explained in the following were obtained from an optimization process and represent the final model used for the analyses discussed subsequently.

3.1.1 Finite element model

The axisymmetric model dimensions are 20 x 41 m and the mesh is discretized with 15-noded triangular elements. The y_{min} and y_{max} displacement boundaries were set to fixed and the

Table 1. Investigated MC-materials

Material	***E* [kPa]**	***v* [-]**	***c* [kPa]**	**φ [°]**	**ψ [°]**	**p_0 [kPa]**
Mat 1	25,000	0.2	0	30	0	50
Mat 3	8,000	0.2	3	27	0	50

x_{min} and x_{max} boundaries were chosen as normally fixed. The initial radius of the cavity a_0 was chosen with 0.1 m. The cavity cluster was devided into four elements. Nine nodes (A to I) and ten stress points (K to T) across the cavity were selected, as suggested by Xu (2007), for the output of total displacements $|u|$ and radial stresses σ_1'. The exact locations of the nodes can be found in Figure 2, next to the illustration of a typical mesh.

3.1.2 Materials

The material within the cavity was modelled linear elastic with a Poisson‘s ratio of $\nu = 0.0$. In general it was found that $E_{cavity} = 1/5\ E_{continuum}$ yields stable and accurate results. The unit weight of the cavity was chosen with $\gamma' = 0$ kN/m³ and the K_0-coefficient with 1.0. Positive volumetric strains were applied to the cavity in each calculation phase to model the expansion. The 1-m-thick dummy layer was also modelled as a linear elastic material. It was exclusively used for the generation of an initial uniform and isotropic stress field. The soil was modelled with different constitutive models. Basic studies were carried out with the Mohr-Coulomb (MC) constitutive law, but calculations were also made with the Hardening-soil model (HS) and the hypoplastic model by von Wolffersdorff (1996). In all cases the unit weight of the continuum was chosen with 0 kN/m³ and K_0 with 1.0.

3.1.3 Calculation Phases

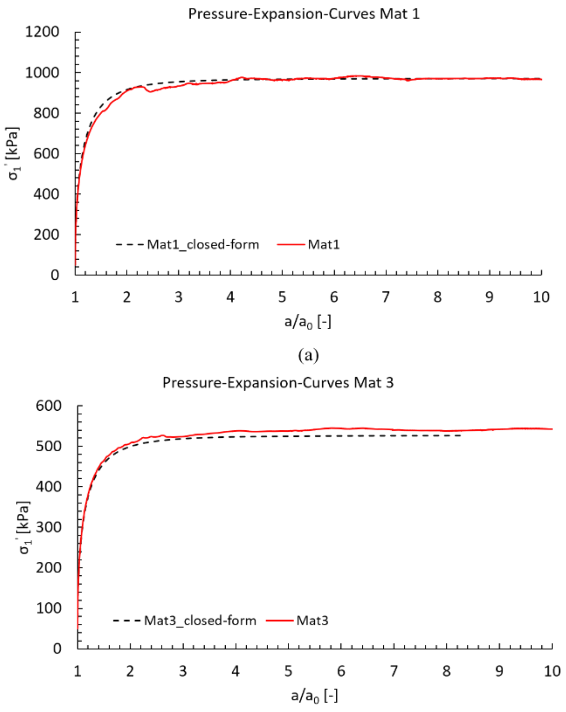
The K_0-procedure was chosen for the initial phase. Within the subsequent expansion phases positive volumetric strains (~100 – 200 % in each phase) were applied to the cavity. The maximum load fraction per step in each phase was reduced from the default value down to a value of 0.005 – 0.01. The tolerated error within the calculation phases was chosen with the default value of 0.01. To allow for a more realistic modelling of the cavity expansion involving large strain deformations, the updated mesh option was selected for all phases. The arch-length control was deselected, as this option generally was found to yield unstable results during the expansion of the cavity. The final-pressure-expansion curves were recieved from averaging the output of the chosen nodes and stress points. This was necessary, especially in case of models with a non-associated flow rule, as the stresses and strains across the cavity developed non-uniformely during the expansion.

3.2 Model verification

The closed-form solution of Yu & Houlsby (1991) was used to validate the FE-model. This analytical solution for the large-strain problem uses the MC constitutive law and offers the option to account for a non-associated flow-rule.

3.2.1 Non-associated flow rule

The studies with the MC-model were based on different materials with soil parameters as can be found in Table 1.



(a)

(b)

Figure 3. Pressure-expansion-curves: Closed-form vs. FE-solution (a) Mat 1, (b) Mat 3

Additionally, the mean effective pressure p_0 is given for which the diagrams in Figure 3 were recieved. Figure 3 shows the pressure-expansion-curves for the considered materials. The ratio a/a_0 is known as the expansion ratio, where a depicts the current cavity radius and a_0 the initial one. The comparisons of the curves from the FE-simulations show a very good agreement with the closed-form solutions acc. to Yu & Houlsby (1991). However, due to the influence of the non-associated flow rule, the recieved curves from the FEA are not totally smooth as the stresses and displacements get distributed unevenly across the cavity during the expansion.

3.2.2 Associated flow rule

Different calculations with $\varphi = \psi$ based on Mat 1 from Table 1 were performed with varying values for the friction angle φ and the dilatancy angle ψ. The values for φ and ψ were chosen in a range between 10 to 30°.

All models using an associated flow rule showed uniform distributions of the stresses and strains across the cavity during the expansion and as a result very smooth pressure-expansion-curves were obtained. It was found that the models with φ and ψ up to 20° were able to accurately reproduce the results of the closed-form solution. In case of higher friction and dilatancy angles the curves from the FE-simulations started to deviate slightly from the reference curves. Figure 4 compares the FE-results of the pressure-expansion-curve to the analytical solution for $\varphi = \psi = 15°$.

3.3 HS-model

The Hardening-soil model was used for further studies with the proposed FE-model. The results for eight different materials found in literature (Xu 2007) were compared to the results of the FE models. The material stiffnesses were given for a pressure p^{ref} of 120 kPa. This is also the value for the effective mean pressure p_0 for which the SCE-simulations were performed. The K_0-coefficient in all cases was given with 1.0. The void ratios e_{max} and e_{min} were used with 0.78 and 0.49 and the initial void ratio for the investigated dense materials was given with 0.5 and with 0.68 for the loose materials. A value of $c = 0.2$ kPa was used for the cohesion and $\nu = 0.2$ for the Poisson's ratio. The unit weight for the soil and the cavity material was assigned with 0 kN/m³ and initial stresses of 120 kPa were generated with a corresponding unit weight of the dummy layer. The remaining soil stiffness and soil strength parameters for 2 materials are given in Table 2. Figure 5 depicts the pressure-expansion-curves received from the SCE-simulations for both materials, as well as the curves given by Xu (2007).

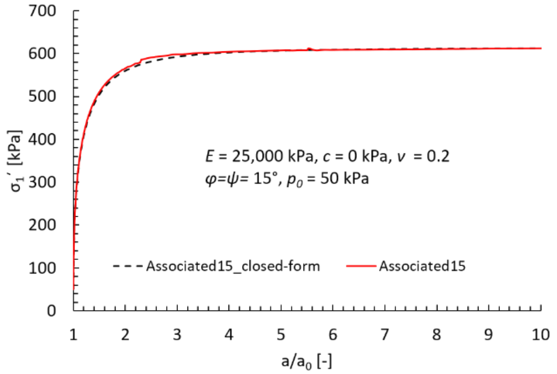


Figure 4. Associated flow rule - pressure-expansion-curves for $\varphi = \psi = 15°$

It can be seen in Fig. 5 that the curves from the PLAXIS calculations follow the once by Xu until the stresses start to rise beyond these curves at certain expansion ratios. It is believed that this is a result of the non-uniform initial stress field that

Table 2. Two of the eight investigated HS materials found in literature (Xu 2007)

Material	E_{50}^{ref} [MPa]	E_{ur}^{ref} [MPa]	E_{oed}^{ref} [MPa]	φ [°]	ψ [°]
HS1 (loose)	10	30	10	40	0
HS2 (dense)	50	150	50	40	0

Table 3. Hypoplastic parameters from material PLM AZ28

Material	φ_c **[°]**	h_s **[MPa]**	n **[-]**	e_{d0} **[-]**	e_{c0} **[-]**	e_{i0} **[-]**	α **[-]**	β **[-]**
PLM AZ28	36.3	39	0.525	0.74	1.261	1.450	0.050	1.97

Xu‘s models were subjected to, as he modeled the continuum with a non-zero unit weight, with p_0 = 120 kPa only at the level of the cavity.

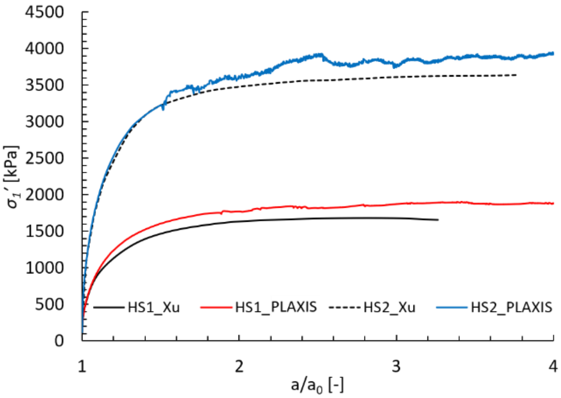


Figure 5. Pressure-expansion-curves of the HS materials found in Table 2

3.4 Hypoplastic model

The „Hypoplastic Sand“ model based on the hypoplastic constitutive law by von Wolffersdorff (1996) was used to conduct KIM-analyses on a material of a land reclamation project in Dubai.

Hypoplastic constitutive laws describe the stress-strain relationship of granular materials with only one single tensorial equation. Therefore, additional formulations used in traditional elasto-plasticity (e.g. the flow-rule) are not required. Hence, the problems of non-uniform stress and strain distributions across the cavity are not encountered within the simulations of the spherical cavity expansion. As a result the recieved pressure-expansion-curves tend to proceed very smooth.

3.4.1 KIM-analysis

A full KIM-analysis was performed for the material PLM AZ28, for which the KIM parameters had already been determined by the lab. The hypoplastic parameters for this material can be found in Table 3. 50 simulations of the spherical cavity expansion were performed at five different initial effective mean pressures p_0 (25, 50, 100, 150, 300 kPa) and ten different values for the initial relative density I_D (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9) to recieve the explicit values for the limit pressure p_{LS}, required to come up with the approximated p_{LS}-curves (to work out the KIM-parameters). Figure 6 gives an example of the recieved pressure-expansion-curves for the five considered effective mean pressures at a relative density of I_D = 0.5. The limit pressures p_{LS} were taken as the maximum values for σ_1' of the recieved pressure-expansion-curves.

From a comparison with the results from the OSINOV code it was found that the limit pressures from the PLAXIS models tend to be slightly smaller than the values obtained from the finite-difference code. The maximum difference was encountered with ~7 %, whereby no dependency on the considered relative densities or effective mean pressures could be observed.

The 50 recieved values for p_{LS} and the approximated p_{LS}-curves from the PLAXIS calculations performed on material PLM AZ28 can be found in Figure 1.

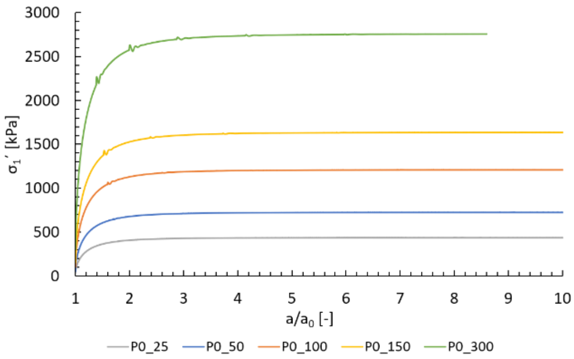


Figure 6. Pressure-expansion-curves for different p_0 at I_D = 0.5 for hypoplastic material PLM AZ28

The differences between the limit pressures recieved from the OSINOV code and the PLAXIS model also resulted in differences in the recieved KIM-parameters, as the curve-fitting procedure is quite sensitive with respect to changing values of p_{LS}. Especially the a_i-parameters seemed to be concerned from the differences in the limit pressures. The values for the KIM-parameters for both calculation approaches can be found in Table 4.

Table 4. KIM-parameter of material PLM AZ28 – PLAXIS vs. OSINOV

Parameter	PLAXIS	OSINOV
a_1	1.950	1.705
a_2	-4.795	-6.083
a_3	-1.492	-1.593
b_1	0.812	0.842
b_2	0.052	0.084
b_3	-1.323	-1.440

The large differences in the KIM-parameters, i.e. a_2 ~ 21 %, however, definitely do not mean that the approximated values for the cone penetration resistance q_c are affected by the same amount as always the whole set of KIM-parameters affect the approximation (see eqns. (1) – (4)).

3.4.2 Sensitivity of the FE model

A sensitivity study with the FE model incorporating the hypoplastic constitutive law was conducted to see how it performs in comparison with the OSINOV code. Based on the studies from Slawik (2018) the material PLM AZ28 (Table 3) was used for this study. The results of the limit pressures from the calculations performed with the FE-model at initial relative densities of $I_D = 0.2$ and 0.8 and an initial mean effective stress of $p_0 = 50$ kPa served as reference values for the comparison. Three factors of the models, namely the domain size, the mesh coarseness and the tolerated error were investigated within the sensitivity study. These model properties can more or less be compared to the boundary radius, the discretization and the time step, which Slawik (2018) varied in his studies with the OSINOV code. Three domain sizes different from the reference models (20 x 41 m) were considered, with 10 x 21 m, 15 x 31 m and 50 x 101 m. The tolerated errors for the study were chosen with 0.001 and 0.025 (reference model - 0.01). The meshes included 431 and 1627 elements (reference model – 740 elements) for the models for which the domain size was not changed from the reference size. For every calculation only one model property was changed, the rest remained the same. The received differences between the reference limit pressure and the limit pressures from the different model variations for $I_D = 0.2$ and $p_0 = 50$ kPa are listed in Table 5.

Table 5. Limit pressures p_{LS} and differences Δp_{LS} received for varying model properties for $I_D = 0.2$

Model	p_{LS} [kPa]	Δp_{LS} [%]
Reference	**565.94**	**-**
Coarse Mesh	570.92	0.88
Fine Mesh	566.77	0.15
Tol. error 0.001	564.97	-0.17
Tol. error 0.025	566.17	0.04
Domain 10x21m	593.11	4.80
Domain 15x31m	569.63	0.65
Domain 50x101m	567.11	0.21

From Table 5 it can be concluded that only the domain size has a significant influence on the resulting limit pressures (in case the dimensions of the models are getting too small). The same behaviour was also observed for the calculations performed at $I_D = 0.8$ and $p_0 = 50$ kPa with a maximum difference from the reference value for a domain size of 10 x 21 m of 5.10 %. To illustrate the sensitivity of both, the PLAXIS and the OSINOV models (for $p_0 = 50$ kPa), diagrams were created which show the maximum deviations of the received limit pressures from the reference limit pressures for the investigated model properties. The graphs regarding the OSINOV code were produced using the results from Slawik's sensitivity study. Figure 7 shows the sensitivity of the PLAXIS and the OSINOV model in terms of maximum differences Δp_{LS} in % exemplarily for $I_D = 0.2$. The factors used for the OSINOV models refer to the suggested calculation parameters for this code. What can be seen from the diagram in Figure 7 is the high sensitivity of the OSINOV code with respect to changes in the calculation parameters. Differences of more than 40 % were observed when the suggested boundary radius was increased by a factor of 4, whereas the PLAXIS model is subjected to a much smaller sensitivity.

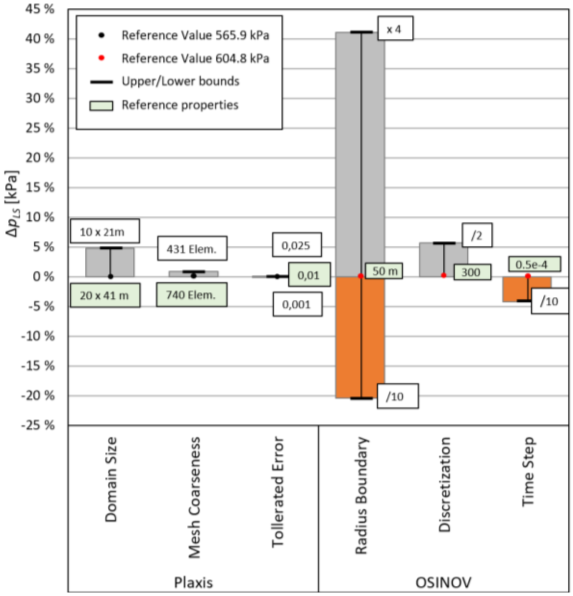


Figure 7. Sensitivity of the PLAXIS and the OSINOV model for $I_D = 0.2$ and $p_0 = 50$ kPa

4 CONCLUSIONS

It could be shown that the finite-element model of the spherical cavity expansion problem can serve as valuable tool for the determination of the developing limit pressures p_{LS}. The recieved pressure-expansion curves for a soil modelled with the MC constitutive law showed a very good agreement with the closed-form solution by Yu & Houlsby (1991). However, it was found that the flow-rule has an impact on the results obtained with the FE-models as in the non-associated case non-uniform stress and strain distributions across the cavity were observed. When using a hypoplastic soil model, the recieved pressure-expansion-curves tend to be very smooth. KIM-analyses using the hypoplastic constitutive law showed that the proposed FE-model tendetially produce limit pressures which are less than 7 % below the limit pressures recieved from the commonly used finite difference code. For the KIM it is especially of importance to accurately describe the mechanical behaviour of the actual materials. Therefore, further research related to the determination of the input parameters for the hypoplastic model, based on in-situ and laboratory tests, is required.

5 REFERENCES

Brinkgreve, R.B.J., Kumarswamy, S. & Swolfs, W.M. 2017. *PLAXIS 2017*. Finite element code for soil and rock analyses, User Manual. Delft: Plaxis bv.

Cudmani, R. 2000. *Statische, alternierende und dynamische Penetration in nichtbindigen Böden.* PhD thesis, Universität Fridericiana Karlsruhe.

H.S. Yu and G.T. Houlsby. 1991. Finite cavity expansion in dilatant soils: loading analysis. *Geotechnique, 41*(2), 173-183.

Meier, T. 2007. *Application of Hypoplastic and Viscohypoplastic Constitutive Models for Geotechnical Problems.* PhD thesis, Universität Fridericiana Karlsruhe.

R. Cudmani and V.A. Osinov. 2001. The cavity expansion problem for the interpretation of cone penetration and pressuremeter tests. *Canadian Geotechnical Journal, 38,* 622-638.

Slawik, S. 2018. *Numerical studies on KIM employing the Cavity Expansion Method.* Master's thesis, Gdansk University of Technology.

Xu, X. 2007. *Investingation of the End Bearing Performance of Displacement Piles in Sand.* PhD thesis, University of Western Australia.