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Pressure drop in vertical pipes of sand boils

Chute de pression dans les chenaux verticaux sous sable boulant

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ABSTRACT: Backward erosion piping is a threat for relatively impermeable dikes that are founded on a sandy, more permeable subsoil. Groundwater flow concentrates on the boundary between the impermeable layer and the sand and may cause erosion of the sand grains. The driving force is the head difference over the dike.
If the sand on which the dike is founded is overlain by a relatively impermeable layer of clay or peat, backward erosion piping is only possible if there is also a vertical flow through a defect in that impermeable layer. The head loss in the vertical defect is of importance since it reduces the head loss over the horizontal pipe. Up to now, a rule of thumb is used to determine this vertical head loss. From a theoretical perspective, the head loss depends on the flow velocity in the vertical pipe and can be between 0**d* and 1**d* (with *d* the thickness of the impermeable layer). A calculation model is presented in this paper to relate the head loss in the defect to the groundwater flow towards the defect.

RÉSUMÉ: Les canalisations d'érosion en amont constituent une menace pour les digues relativement imperméables qui reposent sur un sous-sol sableux, plus perméable. L'écoulement des eaux souterraines se concentre le long de l'interface entre la couche imperméable et le sable et peut provoquer une érosion des grains de sable. La force motrice est la différence de charge hydraulique sur la digue.
Si le sable sur lequel repose la digue est recouvert d'une couche relativement imperméable d'argile ou de tourbe, un chenal d'érosion en amont n'est possible que s'il existe également un écoulement vertical à travers un défaut de cette couche imperméable. La perte de charge dans le défaut vertical est importante car elle réduit la perte de charge sur le chenal horizontal. Jusqu'à présent, une règle empirique est utilisée pour déterminer cette perte de charge verticale. D'un point de vue théorique, la perte de charge dépend de la vitesse d'écoulement dans le chenal vertical et peut être comprise entre 0**d* et 1**d*. Un modèle de calcul est présenté dans cet article pour relier la perte de charge dans le défaut à l'écoulement des eaux souterraines vers le défaut.

Keywords: Backward erosion piping, analytical model, defect, vertical pipes, levee.

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1 INTRODUCTION

Backward erosion piping is a dangerous failure mechanism for relatively impermeable dams and levees founded on a permeable subsoil. This situation is quite often present along rivers and river deltas. Examples are the Mississippi river (USACE, 1956; Glynn et al., 2012), The Netherlands (Van Beek, 2015), and the Po river area in Italy (Gracía Martínes et al., 2017). In these countries, backward erosion piping is recognized as the number 1 or number 2 most dangerous failure mechanism for river levees.

If the sand on which the dike is founded is overlain with a relatively impermeable layer of clay or peat, backward erosion piping is only possible if there is also a vertical flow through a pipe or crack in that impermeable layer; see Figure 1.

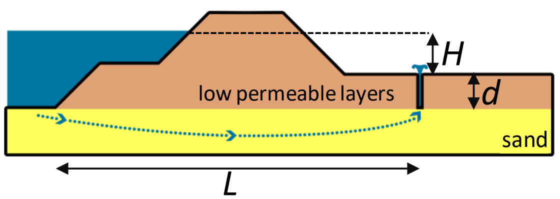

Figure 1. Sketch of a levee prone to backward erosion piping with a vertical crack in the low permeable top layer.

Whether or not piping occurs depends on various factors such as the grain size and the grain size distribution of the sand and the geometry of sand and impermeable layers, but a very important factor is the overall gradient in the sand, defined as the head loss over the levee (H) divided by the length of the flow path to the vertical crack (L). For example, the piping rule by Bligh (1915) states that for fine sand, L/H should be larger than 18. Although the piping rule of Bligh is abandoned in the Dutch safety assessment (Van Beek, 2015), it still represents the order of magnitude of differential head required to cause piping. With this rule it can be deduced that the head loss in the vertical crack is quite important to determine the risk on piping. The head loss in the defect reduces the head loss across the aquifer, and therefore, according to Bligh's rule, every meter of head loss will reduce the design seepage length by 18 meters.

The vertical gradient has been used as part of piping assessments for decades. In the U.S., where only initiation is assessed for design, 0.5d has been used as an empirical criterion for sand boil initiation (USACE, 2000). In the Netherlands, where progression is assessed, 0.3d has been assumed as the vertical gradient over a vertical sand boil pipe (TAW, 1999). Once a sand boil has activated, the head loss in the sand boil is not a constant (Robbins et al., 2018), but rather a function of sand characteristics, opening dimensions, and flow rate. The current paper explores how the head loss in the vertical sand boil pipe varies.

The Dutch rule is based on fluidisation experiments (Yap, 1981), and the USA rule is based on field experiences, where it was found that sand boils hardly ever occur when the vertical gradient was less than 0.5 (corresponding to a head difference of 0.5d)(Ammerlaan, 2007).

2 RECENT RESEARCH

Recently, Bezuijen (2015) developed a model describing the possible head drop in a vertical pipe using the theory of hindered settling. He assumed a vertical defect filled with a sand-water mixture of a concentration c and a half spherical erosion hole in the sand. It is assumed that the hydraulic head is known at some distance from the erosion hole in the sand. Robbins et al. (2018) compared the field measurements of the head loss with a slightly different theory but also based on hindered settling in a vertical pipe. They measured the discharge, velocity, and hydraulic gradient over a vertical pipe. In both models, it is assumed that for a certain outflow velocity, the concentration of the sand grains is limited by the settling velocity of the grains at that concentration. In case the outflow velocity is larger than the fall velocity of a single grain in the pipe, the gradient will be zero. At low outflow velocities, it is possible that a sand-water mixture with a certain concentration of sand grains exists in the vertical pipe, and a higher gradient is possible. In both situations, head loss may occur in the top of the defect, due to the sand boil activity itself. In their comparison, they found reasonably good agreement between measurements and theory, both for the field situations and experiments by Yap (1981). The results show that in the experiments, vertical gradients (d*h*/dz) between 0.3 and 0.9 were measured. In the field tests, a value close to 0.1 in one test and 0.62 in the other was obtained.

The theory presented by Robbins et al. (2018) assumes that the flow velocity is known. This could be done because this flow velocity was measured in the field tests. Bezuijen (2015) assumes that the hydraulic head at some distance of the outflow point is known. When designing a new levee or assessing the stability of an existing levee, such a flow measurement or hydraulic head measurement is not available, but it is necessary to calculate the flow velocity. The next sections present such a calculation based on an analytical model presented by Bezuijen (2017).

3 PRINCIPLE

The aim of the calculation model is to predict the pressure drop in the vertical branch of the pipe as a function of the flow velocity of the water supplied by the aquifer and the corresponding particle concentration.

A limited flow through the defect is possible when the concentration of grains is large, resulting in a relatively large pressure drop. If the flow in the pipe increases, the concentration of grains in the pipe will decrease, and thus the pressure drop will also decrease. This is illustrated nicely by column experiments performed by Yap (1981). He used a vertical column filled with sand and increased the water discharge from below. Initially the sand sample is still intact and the gradient increases linearly with the flow velocity (Darcy flow).

At a gradient of approximately one, there is no effective stress anymore, and the sample changes to a sand-water mixture. When the discharge is further increased, the total pressure at the lower end of the column remains constant, and the pressure gradient decreases while the length of the sand-water mixture in the column increases with increasing discharge (which corresponds with a lower density of the mixture since the total amount of sand in the column remains the same). Yap (1981) also showed that the roughness of the pipe walls hardly influences the result.

In the case of backward erosion piping, the flow in the vertical pipe comes from the horizontal pipe in the sand bed. The aquifer resistance may limit the quantity of groundwater flow that passes through the vertical pipe. Then, the concentration in the pipe will not decrease below a certain value. This will be quantified for a simplified situation in the following sections. This simplified situation shows the capabilities of this way of modelling. In further research, it will be advisable to use numerical modelling for situations that come closer to real situations of backward erosion piping.

4 CALCULATION METHOD

Consider a dike section with a vertical pipe, and a width of B m, or with a piping hole every B m for an ongoing section (Figure 2). The dike is placed on a sandy aquifer with a thickness of D m and a permeability of k m/s. The top layer on the landward side of the dike is a low permeable layer with thickness d m. The seepage length is L m. The dike is assumed impermeable over this L m. On the landward side, the aquifer is semi-confined with a leakage length of λ m. The leakage length is defined as $\lambda = \sqrt{kdD/k'}$, where k' is the permeability of the low permeable layer with thickness d. The head difference over the levee is H m; see also Figure 1. The vertical pipe has a radius of r_0 m, and due to erosion, it is assuremed that there is a semi-spherical hole below the vertical pipe with a radius of $a.r_0$; see Figure 3. A fluidized sand bed with concentration c is assumed in the vertical pipe.

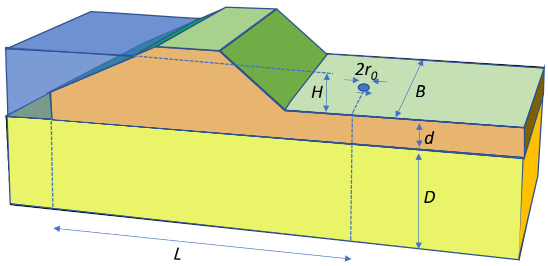

Figure 2. Definition sketch section of dike with piping hole.

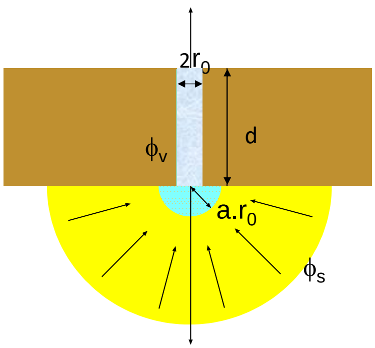

Figure 3. Detail, vertical flow to the exit hole. The sand-water mixture in the vertical pipe has a sand concentration (c).

The fall velocity of grains w in m/s for hindered settling in a fluidized bed is given by Richardson and Zaki (1954).

$$w = w_0(1-c)^n \tag{1}$$

where, w_0 is the fall velocity of a single grain (m/s), c the volumetric concentration of the grains, and n an exponent depending on Reynolds number. For fine sand with a d_{50} of 150 μm, as used in this study, this is around 4. In case of laminar flow, w_0 can be described with Stokes' formula:

$$w_0 = \frac{1}{18\mu}(\rho_g - \rho_w)\, g\, d_{50}^2 \tag{2}$$

Where μ is the kinematic viscosity (Ns/m^2), ρ_g and ρ_w are the density of the grains and of the water respectively (kg/m^3), and g the acceleration of gravity (m/s^2).

It is assumed that the sand water mixture in the vertical pipe has a constant concentration (c) along its length, and in the spherical hole in the sand below the pipe, the concentration is zero. For equilibrium in the vertical pipe the specific discharge (v_v) should be equal to the hindered settling fall velocity, or

$$v_v = w \tag{3}$$

Neglecting losses due to flow (Robbins et al. 2018, have shown that these are small), the head drop over the vertical pipe can be written as:

$$\phi_v = \Delta c d \tag{4}$$

In Eq. (4), it is assumed that the hydraulic head is zero on top of the defect. Combination of Eq. (1), (3), and (4) leads to a relation between the vertical velocity and the concentration as

$$v_v = w_0 (1 - \frac{\phi_v}{\Delta d})^n \tag{5}$$

The flow in the vertical pipe should be the same as the flow from the sand. Assuming semi-spherical lines of equal head, the total discharge from the semi-sphere can be written as

$$Q = 2\pi k a r_0 (\varphi_s - \varphi_v) \tag{6}$$

where: a is defined in Figure 3. Continuity requires:

$$v_v = \frac{2.k.a.(\varphi_s - \varphi_{v)}}{r_0} \tag{7}$$

which leads with Eq. (5) to

$$(\varphi_s - \varphi_v) = \frac{r_0.w_0}{2.a.k}\left(1 - \frac{\varphi_v}{\Delta.d}\right)^n \tag{8}$$

Continuity demands that all water that flows underneath the levee flows through the vertical pipe or in the semi-confined aquifer on the landward side of the pipe. With Darcy's law this leads to (see also Bezuijen, 2017)

$$\frac{kBD}{L}(H - \phi_s) = \pi r_0^2 v_v + \frac{kBD}{\lambda}\phi_s \tag{9}$$

where B, D, L and H are defined in Figure 2. By rewriting equation 9, it is possible to calculate the head difference H as shown in

$$H = \left(1 + \frac{L}{\lambda} + \frac{2\pi L a r_0}{BD}\right)\phi_s - \frac{2\pi L a r_0}{BD}\phi_v \tag{10}$$

For a given geometry, as presented in Figure 2 and Figure 3, it is now possible to calculate the hydraulic head H over the structure for different values of the hydraulic head in the vertical pipe ϕ_v or different values of the gradient over the pipe i_{pipe}. When a higher hydraulic head is measured over the total levee structure, this means that the velocity in the vertical pipe is higher than the fall velocity and there will be no equilibrium, sand will be washed out, and the hydraulic gradient in the vertical pipe will decrease. It will be shown that in some cases, H will also decrease when i_{pipe} decreases, which means that there is no equilibrium, and i_{pipe} will become zero (the flow velocity will be larger than the fall velocity of a single grain). For such a situation, there is thus no head difference in the vertical pipe (apart from flow losses that are neglected in this paper).

5 EXAMPLE CALCULATIONS

5.1 Equilibrium calculations

Three cases will be presented. One is in Boretto, Italy, that was also described in Bezuijen (2017) and Gracía Martínes et al. (2017). The second case is in the Netherlands, namely the dike at Oudeschild. Not all parameters necessary for the model were available here, so some have been estimated. The third situation is the same as for Oudeschild, but with a much shorter leakage length, to show the influence of the leakage length. The parameters used in the calculations are summarized in Table 1. In the calculations r_0 was varied from 0.05 to 0.5 m. Figure 4 shows the results for Boretto, Figure 5 for Oudeschild, and Figure 6 for Oudeschild with a short leakage length. In all calculations *a* see Figure 3 is 20. This value is taken assuming that there is a considerable pipe but is just a guess. Field measurements are needed to make a better estimation of this parameter. All calculations show the same pattern: For a small diameter, vertical pipe, the pressure drop in the sand is only small, and the pressure drop H is determined by the pressure drop over the vertical pipe. For large vertical pipes (r_0=0.2 and 0.5 m), the flow resistance in the sand becomes important. As described in Section 4, The calculation method assumes a half-spherical hole in the sand under the vertical pipe (see Figure 3). However, when backward erosion piping occurs, there will be an erosion pipe instead of such a half-spherical hole. Based on a limited amount of numerical calculations, it was found that the discharge to a pipe and such a half-spherical hole is comparable when the circumference of the pipe is 1.3 times longer than that of the half-sphere. This ratio will not be constant for different geometries and leakage lengths, and further study will be necessary.

Table 1. Parameters used in calculations.

parameter	Boretto	Oudeschild	Oudeschild s*	dim.
H_{design}	6	5.6	5.6	m
L	180	125	125	m
B	100	100	100	m
D	30	20	20	m
d	7	4	4	m
λ	1400	250	10	m
d_{50}	150	250	250	µm
k	3*10-5	10^{-4}	10^{-4}	m/s
a	20	20	20	-

* s stands for short leakage length

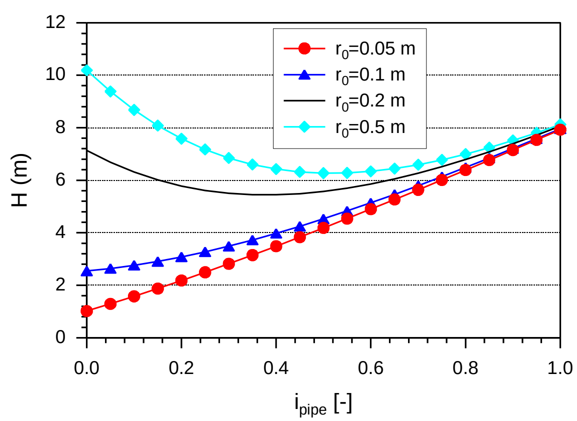

Figure 4. Boretto. Possible hydraulic head as function of the hydraulic gradient in the vertical pipe.

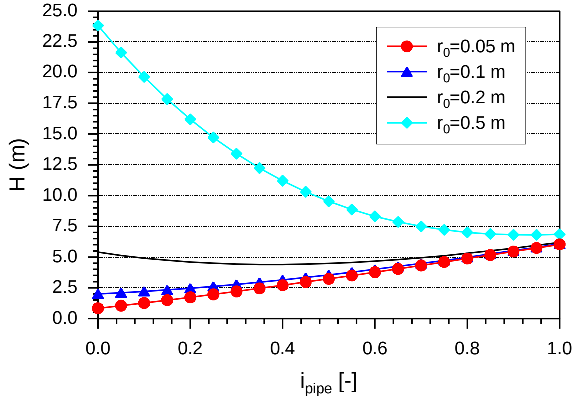

Figure 5. Oudeschild. Possible hydraulic head as function of the hydraulic gradient in the vertical pipe.

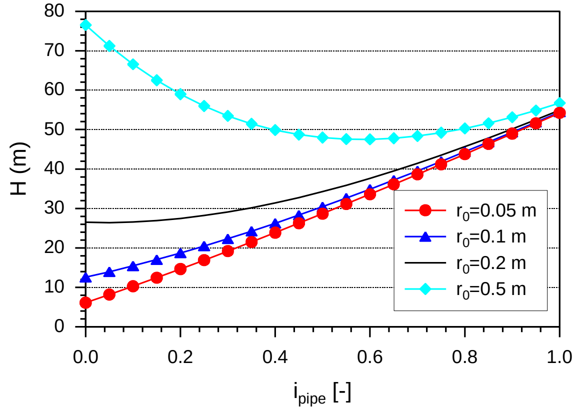

Figure 6. Oudeschild short leakage length. Possible hydraulic head as function of the hydraulic gradient in the vertical pipe.

5.2 Non-equilibrium calculations

In situations where the hydraulic head is larger than calculated in the previous section, erosion will occur. For such situations, it is possible to make an upper-bound calculation on the amount of erosion that can be expected. The equations presented in Section 4 are used to calculate H, ϕ_s, and ϕ_v in an equilibrium situation where the settling velocity is equal to the flow through the pipe. When the applied hydraulic head (H_a) is higher, then for a certain concentration c in the vertical pipe, ϕ_v, will be the same (since in this model ϕ_v is determined by the concentration only), and the hydraulic head difference over the sand will be H_a-ϕ_v. In equilibrium condition, this was H-ϕ_v. Darcy flow is assumed in the sand, and therefore the flow in the sand, and in the vertical pipe will be $(H_a-\phi_v)/(H-\phi_v)$ times the flow found in the equilibrium situation. Now there will be no equilibrium, and sand will be transported continuously through the pipe. Since the velocity in the pipe is known as well as the fall velocity, it is possible to calculate the sand transport S in m^3/s for various concentrations in the pipe.

$$S = \pi r_0^2 c v_f \tag{11}$$

In this upper-bound calculation, it is assumed that the concentration in the pipe is the concentration that allows maximum erosion. In reality, this will be determined by the erosion function.

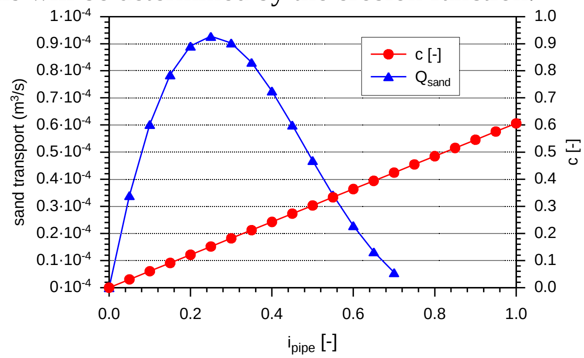

Figure 7. Calculated sand transport for different pipe gradients, for the parameters of Boretto in Table 1, r_0=0.05 and an applied hydraulic head H_a of 6 m. See also text.

An example of this calculated sand transport is shown in Figure 7 for a hydraulic head H of 6 m over the levee. The calculated amount of sand is the in-situ amount assuming an original porosity of 0.4. The maximum transport is reached at a vertical gradient in the pipe of 0.25 and a sand concentration of 0.15 As shown in Figure 4, for i_{pipe} larger than or equal to 0.8, the calculated equilibrium Head H is 6 m or more, therefore, there is no sand transport for these values of i_{pipe}.

Assume again for the Boretto situation, that a vertical pipe with a radius r_0 of 0.05 m and a bit larger hole in the sand (a=2) starts to erode under a hydraulic head H of 6 m. With the assumptions described before, it is now possible to calculate the amount of sand discharged through the pipe, and therefore, we can calculate how a evolves and thus the erosion radius in the sand. The result of such a calculation is shown in Figure 8.

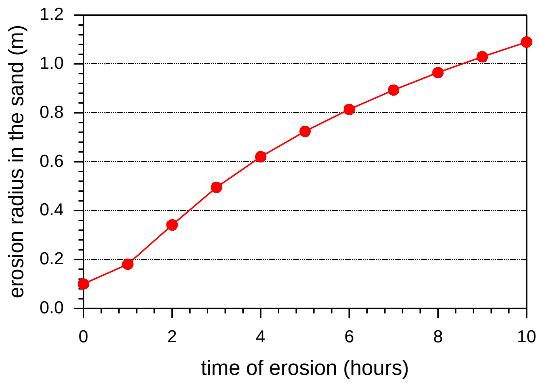

Figure 8. Boretto. Erosion radius in the sand as a function of time.

It appears that after 10 hours of high water, the erosion radius is still only one meter. Assuming a sand boil with roughly a half-spherical shape. this also gives an idea about the dimensions of the sand boil that can be expected. It should be realized that in this calculation, a half-spherical erosion shape is assumed. A thin pipe creating this sand boil can extend over quite a distance.

6 DISCUSSION

For the equilibrium calculations where the curves of H versus i_{pipe} have no minimum, it cannot be guaranteed that there will be sand in a vertical pipe, and thus it cannot be guaranteed that there will be head loss in the defect, such as currently assumed in the rules of thumb. In case a defect is filled, the sand in the defect will contribute to prevent piping, and if the actual head difference measured over the levee is less than the calculated value of H with Eq.(10), there will be no ongoing piping, since the grains will not be washed out the vertical pipe. However, if a defect exists without sand, for example because of dry conditions in the clay upper layer, it will not be filled by the sand from the aquifer because the flow velocity is larger than the fall velocity of a single grain when the applied head is larger than H calculated with Eq. (10) for c=0. In case there is a minimum in the curve, the gradient at which that minimum is reached can be used as a contribution of the stability against piping. Starting with an empty vertical pipe, piping may start, but the flow velocity in the vertical pipe is not large enough to remove the sand from the pipe, and a vertical gradient will remain because of the concentration of sand in the pipe. In all calculated cases where there is a minimum, this was a gradient of more than 0.3. This means that when there is a minimum, the 0.3d rule gives results on the safe side for the situations calculated. In the example calculations, there was only such a minimum for large diameter vertical pipes (r_0 is equal to or larger than 0.2 m). Crucial is therefore the flow resistance in the sand. In case of a low flow resistance compared to the flow resistance in the pipe, the water flow will become too fast, and sand will be washed away.

Comparing the calculated H with the design values presented in Table 1 (H_{design}), it appears that in the case of Borretto, only a very large vertical pipe, i.e., r_0 is equal or larger than 0.5 m, will be able to withstand the design load under all circumstances. For a pipe with radius 0.2 m it is, according to the calculations, allowed to use an 0.4d rule, but that only brings equilibrium up to H=5.44 m. Furthermore, it is clear from the calculation results that the leakage factor has a significant influence. A short leakage factor increases the stability against piping for all diameter vertical pipes significantly, as was also found in earlier calculations (Bezuijen, 2017).

The calculation method presented in this paper can be used to calculate whether a vertical gradient can be considered as is done in the Dutch 0.3d rule and an upper bound solution for the possible erosion rate. The calculation does not predict whether ongoing piping occurs. Whether ongoing piping occurs depends on the hydraulic gradient in the sand.

7 CONCLUSIONS

An analytical model to calculate the influence of a vertical pipe on the possibility and the rate of backward erosion piping is presented. From calculations with the model the following conclusions are possible.

- There is no single rule that describes the head loss in a vertical pipe defect. The head loss in a vertical pipe depends not only on *d* but on other parameters as well.
- A vertical pipe or defect contributes to the stability against backward erosion piping when the flow resistance in the sand is comparable to the flow resistance in the pipe. When this is not the case, the flow velocity in the pipe may easily become higher than the settling velocity of a single grain, and all sand will be removed from the vertical pipe. There can still be sand in the defect due to erosion, but this is not an equilibrium situation; there will be ongoing erosion in the sand.
- For small diameter vertical pipes (smaller than 0.2 m radius), an empty crack will not be filled by the eroding sand because the outflow velocity is too high in the examples calculated.
- The leakage length on the landward side has a significant influence on the results.
- The model can be used to estimate an upper bound erosion rate in case no equilibrium is reached. According to this model, it will take hours before a significant sand boil is formed, when using the parameters for a sand boil found in Boretto.

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