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Experimental results and constitutive model for time-dependent behaviour of sands under oedometric compression

Résultats expérimentaux et modèle constitutif du comportement temporel des sables sous compression oedométrique

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ABSTRACT: Oedometric loading data of dry quartz sand with different initial relative densities was analyzed with respect to the time-dependent behaviour, specifically, creep, rate-dependency and stress relaxation. The test results revealed a significant stress and density dependency of the creep-rate, as well as non-permanent changes of the effective stress following sudden strain-rate changes during strain-rate controlled loading. The experiments with a loading of up to 7.5 MPa effective vertical stress were analyzed using the isotache-concept originally developed for soft fine-grained soils. In this concept, the time-dependent behaviour is governed by the viscosity index I_v, defined as the ratio of the coefficient of creep C_α to the compression index C_c. However, our experimental results show that the isotache-concept does not apply to granular materials. Particularly, the viscosity index I_v is not a material constant, but depends on density and stress. In order to describe the creep behaviour of granular soil under one-dimensional compression, a visco-hypoplastic model was modified. In this new model the viscous strain-rate is a function of the current density, the stress and the overconsolidation ratio according to a Norton-type creep law.

RÉSUMÉ: Les données œdométriques de chargement de sable de quartz sec de densités relatives initiales différentes ont été analysées en ce qui concerne le comportement en fonction du temps, en particulier le fluage, la dépendance au débit et la relaxation des contraintes. Les résultats des essais ont révélé une forte dépendance de la vitesse de fluage par rapport à la contrainte et à la densité, ainsi que des changements non permanents de la contrainte effective à la suite de changements soudains de la vitesse de déformation pendant la charge contrôlée de la vitesse de déformation. Les expériences avec une charge allant jusqu'à 7.5 MPa de contrainte verticale effective ont été analysées à l'aide du concept des isotaches développé à l'origine pour les sols à grains fins mous. Dans ce concept, le comportement en fonction du temps est régi par l'indice de viscosité I_v, défini comme le rapport du coefficient de fluage C_α sur l'indice de compression C_c. Cependant, nos résultats expérimentaux montrent que le concept d'isotaches ne s'applique pas aux matériaux granulaires. En particulier, l'indice de viscosité I_v n'est pas une constante de matériau, mais dépend de la densité et de la contrainte. Afin de décrire le comportement de fluage des sols granulaires sous compression unidimensionnelle, un modèle visco-hypoplastique a été modifié. Dans ce nouveau modèle, la vitesse de déformation visqueuse est fonction de la densité de courant, de la contrainte et du rapport de sur-consolidation selon une loi de fluage de type Norton.

Keywords: Sand, Viscosity, Creep, Rate-dependency, Stress-relaxation

1 INTRODUCTION

The mechanical behaviour of soil is time-dependent. Time-dependency is usually referred to as creep – describing the time-dependent development of strains under constant effective stress – and the dependency of the stress-strain-relationship on the strain-rate. The change of stress with time under constant strain – so called stress-relaxation – is a special case of rate-dependency at zero strain-rate.

The time-dependent behaviour of granular soils depends on the applied effective stress, the density, the mineralogy hence grain hardness and strength, the grain shape e.g. surface roughness and the degree of saturation. It is directly related to the amount of particle degradation. Time-dependent particle degradation and breakage is considered to be the key mechanism behind creep in granular materials (McDowell, 2003, Kwok and Bolton, 2013, Michalowski *et al.*, 2018).

Contrary to that, creep in fine-grained soils can be described by a thermally activated process. Buisman (1936) discovered that the creep-rate $\dot{\varepsilon}$ of fine-grained soils in oedometric testing can be described by

$$\dot{\varepsilon} = \frac{C_B}{t} \tag{1}$$

with C_B the creep coefficient and t the creep time.

Šuklje (1957) recognized that after completion of consolidation under oedometric conditions, stress-strain-curves of equal strain-rate form a band of parallel curves in the logarithm of stress-void ratio diagram and that these curves are also lines of equal creep time. Based on this isotache-concept the analytical and constitutive description of the viscous soil behaviour was developed resulting in the findings of Klobe (1992) and Krieg (2000) that all viscous effects – creep, rate-dependency and stress relaxation – can be described by the viscosity index I_v through

$$\left(\frac{\sigma'_0}{\sigma'_i}\right) = \left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_i}\right)^{I_v} = \left(\frac{t_i}{t_0}\right)^{I_v} = \left(\frac{t+t_0}{t_0}\right)^{I_v}, \tag{2}$$

with relaxation

$$\left(\frac{\sigma'_0}{\sigma'_i}\right) = \left(\frac{t_i}{t_0}\right)^{I_v}, \tag{3}$$

rate-dependency

$$\left(\frac{\sigma'_0}{\sigma'_i}\right) = \left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_i}\right)^{I_v} \tag{4}$$

and creep

$$\Delta\varepsilon = C \cdot ln\left(\frac{t+t_0}{t_0}\right) = C \cdot ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_i}\right). \tag{5}$$

This approach is based on the strain history and strain-rate history independence, which was proven for fine-grained soils by Krieg (2000). It furthermore assumes that $I_v = C_\alpha / C_c$, with C_α the creep coefficient based in the void ratio and C_c the compression index.

It will be analysed in this study to what extend the presented approach is applicable for the description of viscous behaviour of granular soils. Furthermore the effect of stress and density dependent creep will be introduced into a one-dimensional hypoplastic constitutive model.

2 MATERIALS, SPECIMEN PREPARATION AND TESTING PROCEDURE

The used soil was a medium sand (MS) from the Rhenish lignite mining area near Cologne (Germany) with subangular to rounded particles.

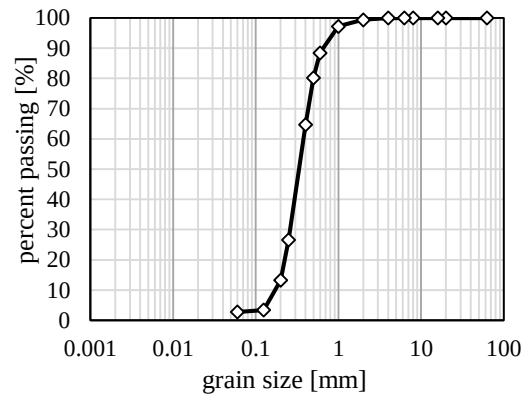

Figure 1. Grain size distributions of sand MS

The sand was uniform and poorly graded. The mineralogical components were 97 % quartz and 3 % orthoclase.

Figure 1 shows the grain size distribution and Table 1 summarizes the classification characteristics of sand MS.

Table 1. Classification characteristics of sand MS

C_u	C_c*	d_{50}	e_{min}	e_{max}
2.11	0.99	0.340	0.628	1.088

The sample size in the standard oedometer devices was 100 mm in diameter and 20 mm in initial height. Dry funnel deposition with air-dry samples was used for specimen preparation. All specimens were carefully levelled with a steel ruler. With this preparation technique minimum relative densities of about $I_d = 0.3$ (loose) were achieved. For the preparation of dense samples the specimens were compacted by laterally tapping the side of the oedometer ring with a hammer in a symmetrical pattern. The prepared maximum relative density was $I_d = 1.0$ (very dense).

The tests were performed using a computer-controlled electromechanical load-frame driven by a stepper-motor. The vertical displacement of the specimens was measured with a resolution of 0.025 µm. All tests were performed under temperature controlled conditions with a maximum temperature fluctuation during one day of 0.5 °C. A possible machine influence on the stress-strain curve due to inertia upon strain-rate change and a possible stress-relaxation resulting from the load-frame were evaluated by loading cap springs of hardened steel and found to be negligible.

2.1 Experimental program

Three different test series were performed on sand MS:

1. *Creep tests* with creep after constant rate of strain at varying density and creep after different strain-rates,
2. *strain-rate tests* with constant rate of strain and sudden changes in strain-rate and
3. *relaxation tests* with relaxation after constant rate of strain at varying density and relaxation after different strain-rates.

3 CREEP TESTS

3.1 Creep after constant rate of strain

The creep tests were performed at a medium dense and very dense initial state. The specimens were loaded with 1.0 %/min strain-rate to vertical effective stresses of 250, 1000, 4000 and 7500 kPa. At each stress level the soil was left to creep for 24 hours. The test results are presented in Figure 2 and Figure 3.

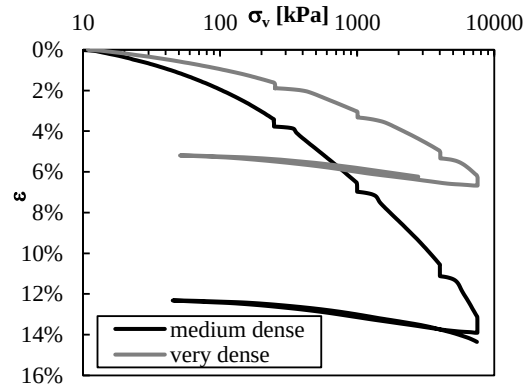

Figure 2. Stress-strain curves at 1.0 %/min at medium dense and very dense state

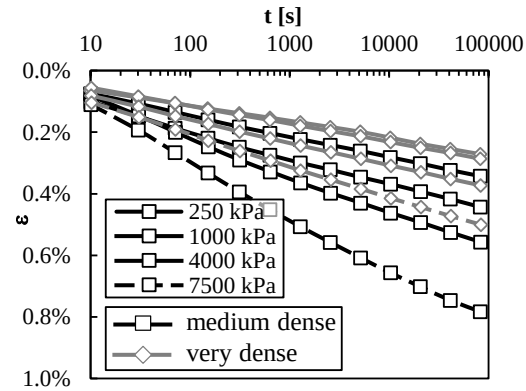

Figure 3. Creep after constant rate of strain of 1.0 %/min at medium dense and very dense state

All creep curves follow a linear – in some cases a slightly under-linear – trend with respect to the logarithm of time. The development of creep strains is clearly stress and density dependent, showing less creep at low stresses and high densities. The linear trend justifies the description of creep strains by Equation (1) at each individual stress and density state. A constant creep coefficient, as it is applicable for normal-consolidated fine-grained soils over a wide stress range, is not valid for sands.

The C_α / C_c ratios evaluated from the creep stages showed a decrease with rising stress and an increase with rising denisty (cf. Figure 4). If only the C_α / C_c ratio is used as coefficient for the description of the viscous behaviour as in Equation (2), this contradicts the behaviour witnessed in the creep tests, that the viscous response increases with rising stress and lower density.

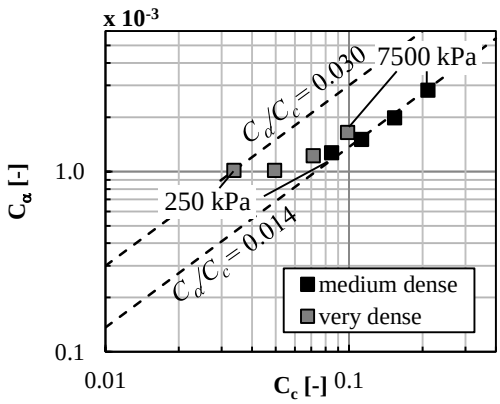

Figure 4. C_α / C_c ratios at medium dense and very dense state between 250 and 7500 kPa

3.2 Creep after different strain-rates

Since Equation (1) is applicable at each individual stress and density state of the sand, the loading strain-rate must have an influence on the initial strain-rate at the beginning of the creep phase and the reference time t_0, marking the point in time where the creep curve calculated by Equation (5) intersects the zero strain axis. The strain-rate at initiation of creep has to be equal to the prior loading strain-rate.

To analyse the influence of the loading strain-rate, tests at medium density were performed with different loading strain-rates prior to 24 hour creep phases. The load-steps were the same as used in the creep tests presented in Section 3.1, at strain-rates of 1.0, 0.1 and 0.01 %/min. Representative for all examined stresses the creep strains at 7500 kPa are displayed in Figure 5. Qualitatively the creep strains at the other stresses showed the same results. As expected, with increasing loading strain-rate the reference time decreases according to Equation (1), which is equivalent to a faster initial creep-rate. As the creep coefficient, also the reference time depends on stress. With increasing creep time, the strain curves converge towards the same slope at each individual stress level, showing that the loading strain-rate has no influence on the long term strain-rate during creep.

It can be concluded that the examined sand shows a dependency of the creep-rate on the loading strain-rate prior to a creep phase. The interrelation can be approximated well by Equation (1).

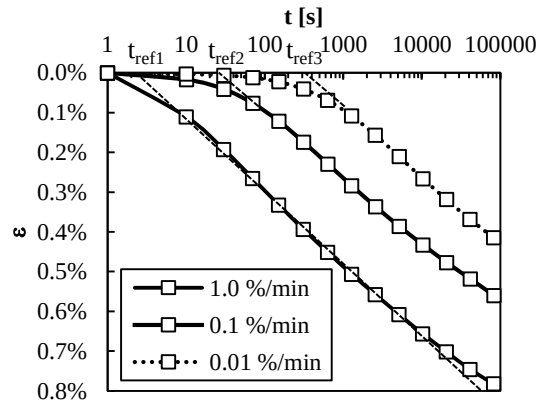

Figure 5. Creep strains after different loading strain-rates at 7500 kPa vertical effective stress

4 STRAIN-RATE TESTS

Studies on the behaviour with sudden changes of strain-rate by Tatsuoka *et al.* (2008) and Enomoto *et al.* (2009) show very diversified

behaviours depending on the material properties. They were categorized as follows (cf. Figure 6):

- Isotache behaviour (as in fine grained soils),
- Temporary Effect of Strain-rate and Acceleration behaviour (TESRA),
- combined behaviour of isotache and TESRA-behaviour and
- Positiv-Negativ behaviour (P&N).

Poorly graded quartz sands like MS mostly show TESRA behaviour (Enomoto *et al.*, 2009).

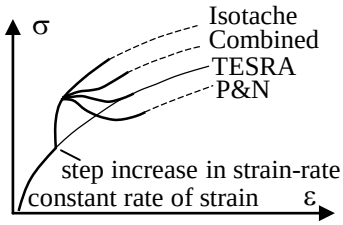

Figure 6. Rate dependent stress-strain responses of granular soils following a sudden change of strain-rate (after Tatsuoka et al., 2008)

4.1 Constant rate of strain tests

The performed constant rate of strain tests on medium dense sand MS show no apparent rate dependency of the stress-strain response (cf. Figure 7). Consequently for sand MS to each pair σ' and e an indefinitely large number of strain-rates could theoretically be assigned. A unique relationship between σ', e, and $\dot{\varepsilon}$ during creep and strain-rate controlled tests is therefore not valid for sands.

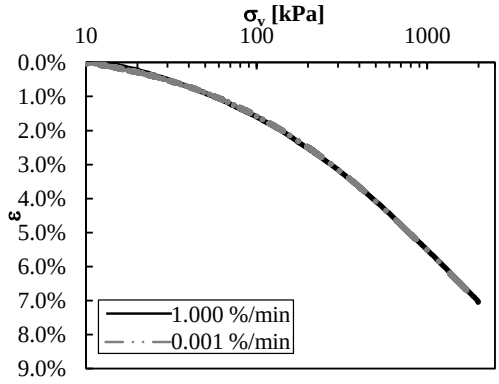

Figure 7. Constant rate of strain tests

4.2 Tests with sudden changes of the strain-rate

The tests with 1000-fold sudden changes of strain-rate during compression show the behaviour presented in Figure 8. Upon deceleration stress decreases immidiately. The initial relaxation rate is faster than the strain-rate of 0.001 %/min. With continued straining the decaying relaxation rate becomes slower than the loading strain-rate and the stress change vanishes.

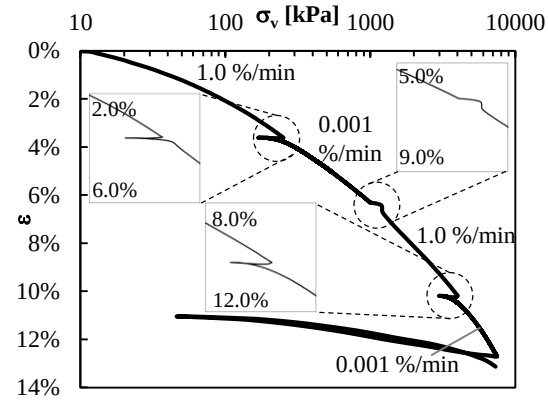

Figure 8. Stress-strain curve with sudden 1000-fold changes in strain-rate

Compression continues as if no strain-rate change had happend. Upon acceleration the stress increases before quickly decaying to the slope of the stress-strain curve before the strain-rate change. The test results confirm the findings from the constant rate of strain tests (Section 4.1) that no permanent stress response is present.

5 RELAXATION TESTS

Stress relaxation is much more pronounced in granular soils than creep deformations or rate-effects (Lade *et al.* (2009) and Lade & Karimpour, 2014). The effect of the stress level and the density as well as of the loading strain-rate before relaxation will be analysed, with the same testing procedure as presented in Section 3.

5.1 Relaxation after constant rate of strain

Figure 9 shows the relaxation behaviour at different effective vertical stresses from 250 to 7500 kPa and at medium dense and very dense initial state. The relative relaxation σ / σ_0, where σ_0 is the effective stress at the start of relaxation, decreases with rising stress. The density seems to have no significant influence on the relaxation behaviour. The I_v values evaluated using Equation (3) decrease from 0.05 at 250 kPa to 0.02 at 7500 kPa.

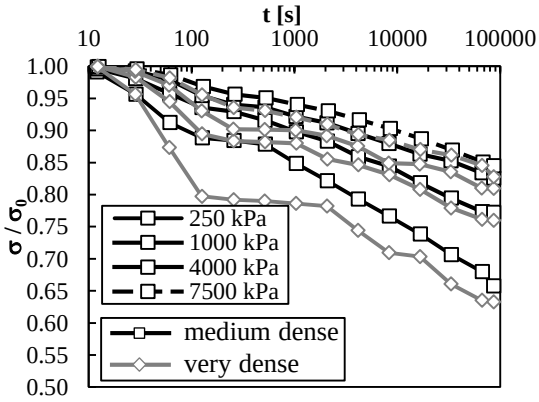

Figure 9. Stress relaxation at different effective stresses and densities after loading with 0.01 %/min

Relaxation shows in this respect qualitatively the same behaviour like the C_α / C_c ratios from creep tests (cf. Section 3.1). The I_v values are about 1.3 to 1.7 times larger than the C_α / C_c ratios, confirming the larger relaxation effect already presented in literature.

5.2 Relaxation after different strain-rates

The influence of the loading strain-rate on the relaxation rate was studied in the same way as presented in Section 3.2 for the creep tests.

Representative for all stresses, the relaxation at 7500 kPa is presented in Figure 10. Qualitatively the relaxation at the other stress levels showed the same results. Like in the creep response, the relaxation reference time increases and the relaxation-rate decreases with lower loading strain-rates, confirming in this respect an isotache behaviour, as analysed in the creep tests.

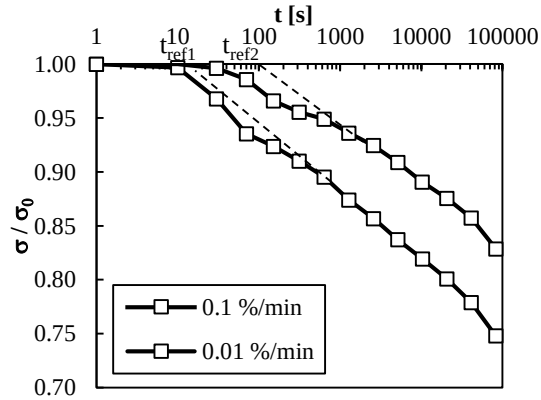

Figure 10. Relaxation after different strain-rates at 7500 kPa vertical effective stress

6 EMPIRICAL CONCEPT FOR THE CREEP COEFFICIENT DETERMINATION

On the basis of serial creep tests on various sands at different densities and stress levels an empirical concept for the density and stress dependent determination of the creep coefficient C_α was developed. It was found that the creep-coefficient increases linearly with the relative void ratio r_e and hyperbolically with the mean stress p' (cf. Figure 11).

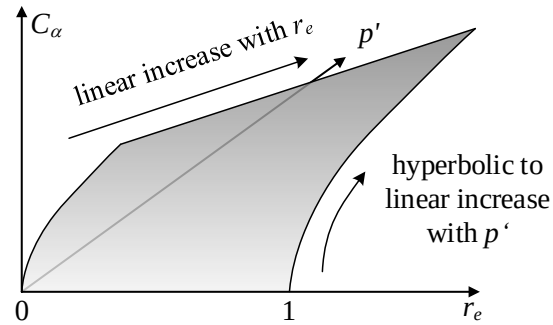

Figure 11. Schematic drawing of creep-coefficient development in the C_α - p' - r_e - space depicted from test results

From that, the following formulas were derived incorporating the stress and density dependent development of the creep potential:

$$C_\alpha = C_{\alpha,ref}\left(\frac{p\prime}{p\prime_{ref}}\right)^\theta \quad (6)$$

$$C_{\alpha,ref} = \omega \cdot r_e + C_{\alpha,ref,0} \quad (7)$$

with $C_{\alpha,ref}$ as reference creep-coefficient at the reference pressure of p'_{ref}. θ as exponent, ω defines the gradient of $C_{\alpha,ref}$ with r_e and $C_{\alpha,ref,0}$ is the creep coefficient at r_e = 0 and p'_{ref}. The approach defines a surface of possible creep coefficients in the C_α - p' - r_e - space. The approach can be calibrated by two oedometer test in a loose and a very dense state, including 24 hour creep phases in the relevant stress range.

7 ONE-DIMENSIONAL COMPRESSION MODEL

As previously presented, the viscous behaviour of sands differs in various aspects from that of fine-grained soils and Equation (2) is not applicable to describe creep, relaxation and rate-dependency in one. To deduce a model that can describe all aspects would lead to an approach, which would be very hard to calibrate and hence its pratical applicability would be doubtful. Therefore, the approach presented here concentrates on the simulation of the creep behaviour, which is most relevant for practical applications.

We consider a one-dimensional compression at constant stress ratio and split the strain-rate into an (non-linear) elastic $\dot{\varepsilon}^e$ and a plastic part $\dot{\varepsilon}^{vis}$ treating all irreversible strains as viscous

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vis}. \quad (8)$$

The stress-rate can be determined via

$$\dot{\sigma} = f_b f_e \cdot \left(\dot{\varepsilon} + f_d \dot{\varepsilon}^{vis}\right) \text{ (Gudehus, 2004)} \quad (9)$$

where f_b, f_e and f_d are the hypoplastic coefficients controlling the stress dependency and the density dependency (von Wolffersdorff, 1996). f_b is rate dependent as by

$$\frac{h_s}{h_{sr}} = 1 + I_v ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right) \text{ (Gudehus, 2004)} \quad (10)$$

with h_s and h_{sr} the granular hardness and reference granular hardness respectively as well as $\dot{\varepsilon}_r = (C_\alpha/(1+e_0))/t_{ref}$ the reference strain rate. C_α can be determined with the approach presented in Section 6. I_v is the viscosity index.

The viscous strain-rate $\dot{\varepsilon}^{vis}$ can be calculated by a Norton (1929) approach, depending on $\dot{\varepsilon}_r$, and $OCR = p_e'/p'$:

$$\dot{\varepsilon}^{vis} = \dot{\varepsilon}_r \cdot exp\left(\frac{1/OCR-1}{I_v}\right)^{1/I_v}. \quad (11)$$

The determination of the equivalent pressure p_e' for calculation of OCR follows the Bauer-compression law (Bauer, 1992) via

$$p'_e = \frac{1}{3} h_{sr}[-ln(e/e_{e0})]^{1/n}. \quad (12)$$

where e_{e0} is the void ratio for p' ≈ 0 on the reference compression line.

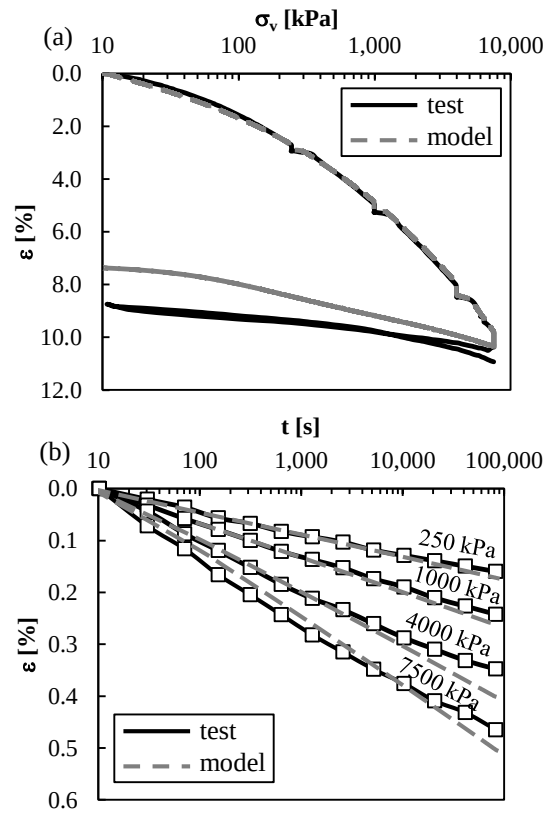

Figure 12. Simulation of a creep test on medium dense sand MS

The stress and density dependent viscosity index $I_v = C_\alpha / C_c$ is calculated with C_α from the approach in Section 6 and the compression index via

$$C_c = e \cdot n \left(\frac{p\prime}{h_s}\right)^n \cdot \ln(10) \text{ (Herle, 1997).} \quad (13)$$

The simulation of a creep test on medium dense MS calculated with $h_s = 1.6 \cdot 10^{-6}$ kPa, $n = 0.285$, $p'_{ref} = 300$ kPa, $C_{\alpha,ref,0} = 10^{-4}$, $\omega = 1.1 \cdot 10^{-3}$, $\theta = 0.3$ and $e_{e0} = 0.876$ is presented in Figure 12. The compression behaviour and especially the stress dependent creep behaviour are well predicted.

8 CONCLUSIONS

Oedometric test results with creep, relaxation and sudden changes of the strain-rate showed that sands display a non-isotache behaviour. The description of all three viscous effects with one parameter I_v is not valid. One-dimensional modelling of the compression behaviour with consideration of the density and stress dependent creep is possible applying the presented approach. Extension of the approach to three dimensions is needed.

9 REFERENCES

Bauer, E. (1992) *Zum mechanischen Verhalten granularer Stoffe unter vorwiegend oedometrischer Beanspruchung*. Universität Fridericiana Karlsruhe.

Buisman, K. (1936) 'Results of long duration settlement tests', in *Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering*. Cambridge, Mass., pp. 103–107.

Enomoto, T., Kawabe, S., Tatsuoka, F., Di Benedetto, H., Hayashi, T. and Duttine, A. (2009) 'Effects of Particle Characteristics on the Viscous Properties of Granular Materials in Shear', *Soils and Foundations*, 49(1), pp. 25–49.

Gudehus, G. (2004) 'A visco-hypoplastic constitutive relation for soft soils', *Soils and Foundations*, 44(4), pp. 11–25.

Herle, I. (1997) *Hypoplastizität und Granulometrie einfacher Korngerüste*. Universität Fridericiana in Karlsruhe.

Klobe, B. (1992) *Eindimensionale Kompresssion und Konsolidation und darauf basierende Verfahren zur Setzungsprognose*. Universität Fridericiana in Karlsruhe.

Krieg, S. (2000) *Viskoses Verhalten von Mudden, Seeton und Klei*. Universität Fridericiana in Karlsruhe.

Kwok, C. Y. and Bolton, M. D. (2013) 'DEM simulations of soil creep due to particle crushing', *Géotechnique*, 63(16), pp. 1–12.

Lade, P. V. and Karimpour, H. (2014) 'Stress relaxation behavior in Virginia Beach sand', *Canadian Geotechnical Journal*, 52(7), pp. 813–835.

Lade, P. V., Liggio, C. D. and Nam, J. (2009) 'Strain Rate, Creep, and Stress Drop-Creep Experiments on Crushed Coral Sand', *Journal of Geotechnical and Geoenvironmental Engineering*, 135(7), pp. 941–953.

McDowell, G. R. (2003) 'Micromechanics of creep of granular materials', *Géotechnique*, 53(10), pp. 915–916.

Michalowski, R. L., Wang, Z. and Nadukuru, S. S. (2018) 'Maturing of contacts and ageing of silica sand', *Géotechnique*, 68(2), pp. 133–145.

Norton, F. H. (1929) *The Creep of Steel at High Temperatures*. New York: Mc Graw-Hill.

Suklje, L. (1957) 'The Analysis Of The Consolidation Process By The Isotaches Method', *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering*, 1, pp. 200–206.

Tatsuoka, F., Di Benedetto, H., Enomoto, T., Kawabe, S. and Kongkitkul, W. (2008) 'Various Viscosity Types of Geomaterials in Shear and Their Mathematical Expression', *Soils and Foundations*, 48(1), pp. 41–60.

von Wolffersdorff, P. A. (1996) 'Hypoplastic relation for granular materials with a predefined limit state surface', *Mechanics of Cohesive-Frictional Materials*, 1(3), pp. 251–271.