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PMMS – Probabilistic Model Macro Stability – with layer boundary uncertainties. General description and example

PMMS - Modèle probabiliste pour la stabilité macro - avec des incertitudes aux limites des couches. Description générale et exemple

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ABSTRACT: In a finite element model, the shear strength reduction method is a generally accepted method to compute a safety factor of geotechnical structures, such as dikes. Nevertheless, a sole safety factor provides insufficient information on the reliability of a dike. Therefore, reliability analyses are needed to assess the actual safety of a dike. This paper puts forward the Probabilistic Model Macro Stability (PMMS), in which the undrained shear strength of soil is modelled through an adapted First Order Reliability Method (FORM) analysis of the SHANSEP model based on log-normally distributed parameters. Besides involving the correlations of the shear strength among different soil layers, the main novelty of this model is the capability to take into account uncertainties in the locations of layer boundaries in the reliability index, using the Stochastic Underground Model [SUM]. In this paper, the model background, input parameters, operation, and a practical application of the PMMS are explained using an example.

RÉSUMÉ: Dans un modèle par éléments finis, la méthode de réduction de la résistance au cisaillement est une méthode généralement acceptée pour calculer un facteur de sécurité des structures géotechniques, comme les digues. Néanmoins, un seul facteur de sécurité ne fournit pas suffisamment d'informations sur la fiabilité d'une digue. La méthode de Monte Carlo est évidemment l'approche la plus appropriée pour effectuer des analyses de fiabilité. Malgré sa précision, l'inconvénient de cette méthode est qu'elle prend trop de temps. De nombreuses recherches ont été consacrées à la recherche de solutions de rechange pour évaluer la fiabilité de la digue d'une manière relativement rapide et facile avec un niveau de précision souhaité. Cet article propose un modèle probabiliste de stabilité macro-macro (PMMS), dans lequel la résistance au cisaillement non drainée du sol est modélisée par une analyse FORM adaptée du modèle SHANSEP. Outre les corrélations de la résistance au cisaillement entre les différentes couches de sol, la principale nouveauté de ce modèle est la capacité de prendre en compte les incertitudes liées à l'emplacement des limites des couches dans l'indice de fiabilité. Dans cet article, le contexte du modèle, les paramètres d'entrée, le fonctionnement et une application pratique du SGGP sont expliqués dans un exemple.

Keywords: probabilistic model; macro-stability; SHANSEP; multivariate statistics; Monte Carlo simulation

1 INTRODUCTION

Geotechnical Macro Stability (of which slope stability is a form) is often the determining factor to design a dike or a dike strengthening and, therefore, its costs. This implies a need for methods to design dikes as economically as possible. This however on the precondition that they meet the required safety level.

One method to achieve this requirement is through a probabilistic design.

The Probabilistic Model Macro Stability (PMMS) is a further development of the probabilistic model presented in (Bakker, 2005).

The objective of the PMMS is to provide a practical, quick, simple to use and reliable model to convert a safety factor for macro stability, for which a two-dimensional finite element model calculation using PLAXIS is applicable, into a Reliability Index. Key features of the PMMS are (also illustrated in Figure 1):

a) The safety analysis of only one or a small number of PLAXIS (FEM but also LEM) calculations is necessary to determine the Reliabity Index of the dike;
b) The SUM – Stochastic Underground Model - which has the capability to take into account the uncertainties of the layer boundaries in order to determine the Reliability Index;
c) The ability to include the uncertainties in the shear strength parameters and their correlations among various soil layers

Figure 1 also shows the simple flow chart of the PMMS which shows that it is a practical method to assess the Reliability Index.

It is stated here that this paper is intended as an introduction to the PMMS and to its possibilities. It is not meant as an exhaustive scientific recording of all procedures implemented.

It is supposed that this will take place in future articles.

This paper describes the basic features of the PMMS, touches on the probabilistic and mathematical theory that is in the model, gives an impression of the input and output and shows the possible impact of including uncertainty of layer boundaries in the calculated Reliability Index β.

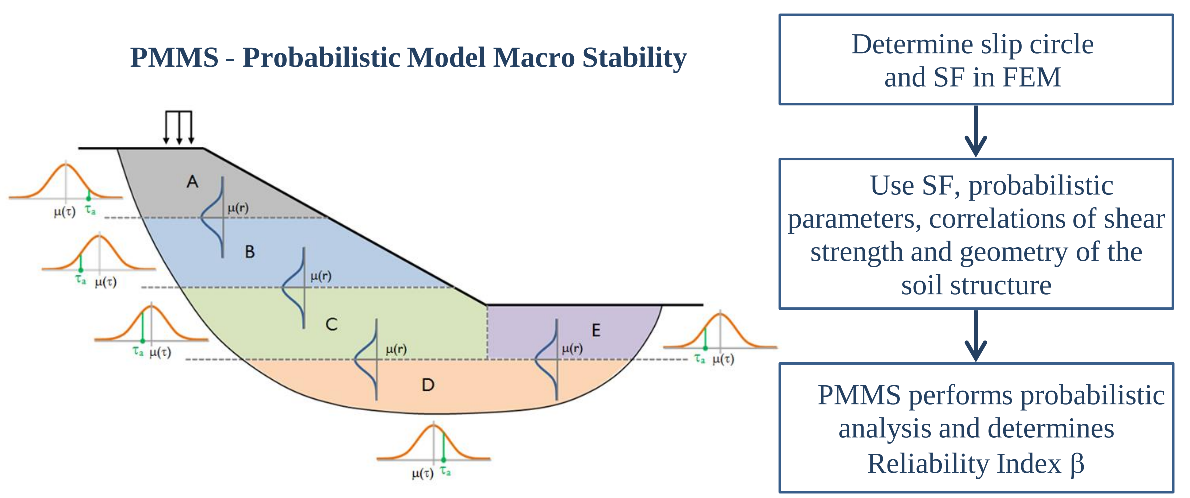

Figure 1 Main principles in the PMMS - Probabilistic Model Macro Stability

2 MODEL DESCRIPTION

The PMMS holds the main principle that the strength distribution and failure mechanism after a strength reduction are a good approximation of the strength distribution and failure mechanism that would be found if the structure in question would collapse under the same load conditions due to insufficient soil strength. This principle can be demonstrated with PLAXIS calculations.

2.1 Adopted shear strength models

In the PMMS both the Mohr-Coulomb model and the Coulomb Friction model are implemen-ted. In this paper the focus is on the SHANSEP (Stress History And Normalised Soil Engineering Properties) concept (Ladd and Foott, 1974).

In the examples given, the shear strength for the drained soil layers is modelled based on the Mohr-Coulomb criterion.

Based at the framework of the Critical State Soil Mechanics (CSSM) (Schofield and Wroth, 1968), the shear strength for drained soil layers can be expressed (simplified) as:

$$\tau = \sigma'_n \sin\varphi' \qquad (1)$$

in which φ' represents the friction angle at the critical state and σ'_n the normal effective stress along the sliding surface.

For the undrained soil layers, the shear strength is modelled in the PMMS according to the SHANSEP concept. According to this principle the shear strength is expressed as:

$$s_u = S\,\sigma'_1\left(OCR\right)^m = S\,\sigma'_1\left(\frac{\sigma'_y}{\sigma'_1}\right)^m \qquad (2)$$

where s_u is the undrained shear strength, S is the undrained shear strength ratio, σ'_1 is the effective major principal stress, OCR denotes the over-consolidation ratio, σ'_y is the effective yield stress, and m denotes the strength increase expo-nent.

The safety factor is determined by reducing the shear strength of the soils until a failure mecha-nism occurs and no further reduction is possible. This method in PLAXIS is known as the Phi/C reduction or Safety Analysis method (Brinkgreve and Bakker, 1991).

In the framework of the CSSM, the safety fac-tor SF is expressed by:

$$SF = \frac{\tau_a}{\tau_c} \qquad (3)$$

where τ_a is the unknown in situ actual shear strength of the soil and τ_c denotes the minimum shear strength needed to maintain the equilibrium.

In the case of undrained soil layers, SF also can be written as the ratio between the actual un-drained shear strength to the minimum undrained shear strength needed to maintain the equilibrium as expressed as follows:

$$SF = \frac{S_u\ input}{S_u\ equilibrium} \qquad (4)$$

2.2 Assessment of shear strengths in under ground model

Soil properties, as determined from small test samples within a layer, usually reveal considera-ble spatial variability within the soil unit co-vered by the test sample set (Bakker, 2005).

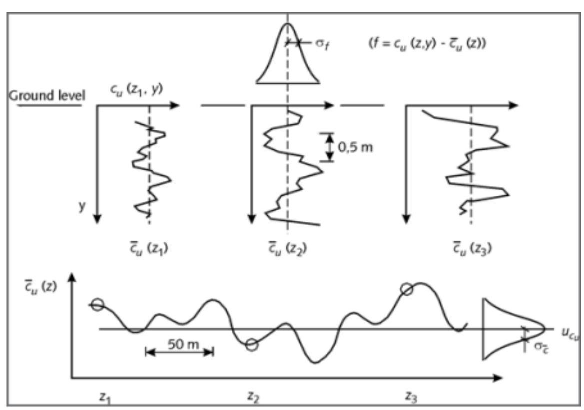

Figure 2 Spatial variability of the shear strength

Figure 2 (upper part) shows that spatial variability within a layer can be present in both horizontal and in vertical directions.
The lower part of Figure 2 shows that averaging of the spatial variability results in layer variations (average values and coefficients of variation) to be taken into account in a stability calculation.

In the PMMS, it is assumed that layer variations in the shear strength are calculated using a procedure that averages spatial variability within a soil unit to layer variations.

2.2 *Averaging shear strength in a multiple layer system*

When a sliding surface cuts through more than one soil layers (or soil clusters), it can be shown that this may have a favourable effect on the reliability index.
Figure 3 shows that when a ground package is intersected by a sliding surface that consists out of more than one in the shear strength indepen-dent soil layers, the probability that SF is smaller than 1.0, is less than when the sliding surface cuts through only one soil layer.
This principle, called “averaging of uncertainties”, results in a reduction of the spread of the safety factor SF present in the soil structure.

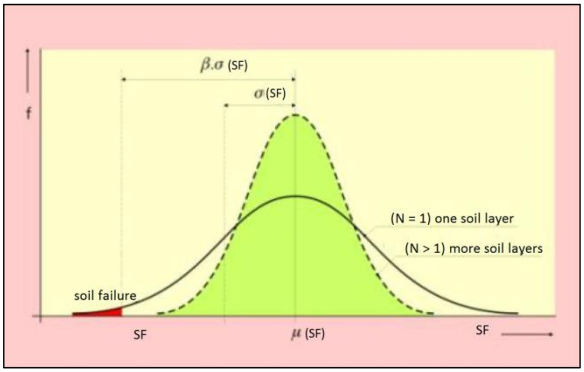

Figure 3 Averaging of uncertainties

However, the extent to which this occurs, depends also on the correlation coefficients of the shear strengths between the different soil layers or clusters to be taken into account. The smaller these are, the smaller the spread of FS becomes.

In general shear strengths are correlated when they belong to the same geological stratum.
In the PMMS they are defined by specifiying a correlation coefficient for each combination of two soil layers. Based on the correlation coefficients for all combinations, the PMMS generates a Shear Strength Correlation Matrix. This matrix is an important part of the Energy Failure Function to be discussed below.

3 CHOLESKY DECOMPOSITION

An important mathematical technique applied in the PMMS is the Cholesky Decomposition of a positive semi-definite symmetrical matrix (Cdt. Benoît, 1925).
The Cholesky Decomposition is an LU decomposition of the form: $A = L \times L^T$ where L is a lower triangular matrix and L^T the transposed matrix of L. In the PMMS Cholesky decompositions serve a twofold purpose:

a) Checking whether a Correlation- or a Covariance matrix represents a physical reality;
b) Converting uncorrelated univariate distributions of the weight factors into a correlated multivariate distribution of the weight factors

4 GEOMETRY

The soil layer geometry in the PMMS is literally the part of the soil layer stratification in which a failure mechanism has developed.
In the example given in Figure 4 this is a sliding surface or shear band. Herein the surface cross section of a shear band element within a layer is the length of the shear band in that layer times a user-defined thickness.
For the correlation indicators, see Section 5.2.

In the PMMS both the surface cross sections of the shear band elements A_Plaxis as follo-wing from the geometry in PLAXIS must be entered, as well as the mean values μ(A) as encountered in the field. See Figure 5.

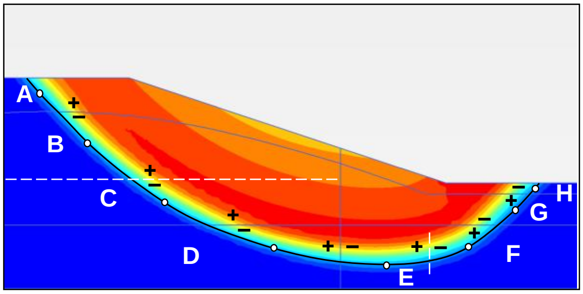

Figure 4 Geometry with failure mechanism and shear band elements, denoted with a letter. The + and – signs are correlation indicators

Further details regarding the geometry are covered in Chapter 5.

Shear band Elements		
Layer ↓	A_Plaxis	μ(A)
A	0,022	0,022
B	0,045	0,045
C	0,045	0,045
D	0,062	0,062
E	0,038	0,038
F	0,038	0,038
G	0,021	0,021
H	0,007	0,007

Figure 5 Shear band elements

5 SUM – STOCHASTIC UNDERGROUND MODEL

The SUM - Stochastic Underground Model - is developed to include geometrical uncertainties into the Reliability Index. Among other mathematical procedures, the SUM algorithm uses multivariate statistics to generate a multivariate distribution of the weight factors of the soil layers in the failure mechanism.
In this chapter the input parameters of the SUM algorithm are discussed.

5.1 Distribution Layer-boundaries

The parameters of the distribution functions of the layer boundaries to be entered into the PMMS are the length L(r) of each layer boun-dary and the standard deviation σ(r)⊥ of the location of it perpendicular thereto. See Figure 6. Besides that, the PMMA needs also the local mutual correlations between the positions of the layer boundaries. See Section 5.2.
As shown in Figure 6, layer boundaries relates to the parts of the layer separations that are located within the faillure mechanism.
In the example considered that is a shear band.

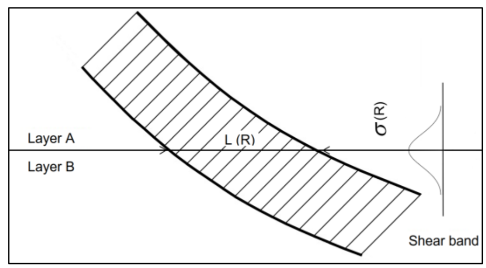

Figure 6 Shear band with layer boundary

The relationship between the layer boundaries (numbers) and the layers (letters) is described with a Layer boundary matrix to be specified to the PMMS. See figure 7.

Layer-boundary Matrix						Layer-boundary →	
Layer ↓	1	2	3	4	5	6	7
A	x						
B	x	x					
C		x	x				
D			x	x			
E				x	x		
F					x	x	
G						x	x
H							x

Figure 7 Layer boundary matrix

5.2 Local Correlations

Each mutual combination of two layer boun-daries has a local correlation coefficient (-1.0 <= $\rho_{i;j}$ <= 1.0) determining to what extent their positions are correlated. “Local” means defined with respect to a not necessarily straight reference axis relating to the considered mutual combination. See Figure 8. Herein “M.C.” denotes a Monte Carlo draw of a layer boundary.

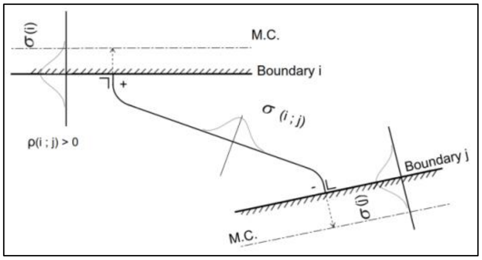

Figure 8 Layer boundary reference axis with + and - signs

Local correlations are positive when in a Monte Carlo draw of the two layer boundaries the length of the reference axis in general increases. If not, they are negative.

This principle should be applied to all mutual combinations of two layer boundaries. In cases that layer thicknesses and its variations are prescribed, $\rho_{i;j}$ can be calculated solving it from:

$$\sigma_{i;j} = \sqrt{\sigma_i^2 + \sigma_j^2 + 2\,\rho_{i;j}\,\sigma_i\,\sigma_j} \qquad (5)$$

Where σ_i and σ_j are the standard deviations of the layer boundaries and $\sigma_{i;j}$ the standard devia-tion of the length of the reference axis.

Only local correlations unequal to zero have to be entered into the input table "Local Correlations" of the PMMS, see Figure 9. Local correlations equal to zero not need to be specified.

	Rho_min	Rho(i;j)	Rho_max	Type
1 en 6	-	-	-	-
2 en 6	-	-	-	-
3 en 6	-1,000	0,750	1,000	B
4 en 6	-	-	-	-
5 en 6	-	-	-	-
	Rho_min	Rho(i;j)	Rho_max	Type
1 en 7	-1,000	0,500	1,000	B
2 en 7	-	-	-	-

Figure 9 Part of Local Correlations table

Not all values of local correlations are physi-cally possible. The PMMS generates a lower - ρ_{min} and upper ρ_{max} limit that they have to comply with. See Figure 9.

For correlations unequal to zero, the PMMS needs also a Correlation Type < A > or < B > to be entered. To that end the PMMS generates a so called "Plus and Min Matrix", see Figure 10.

Pluss and Min Matrix						Layer-boundary →
Layer ↓	1	2	3	4	5	6	7
A	+						
B	-	+					
C		-	+				
D			-	+			
E				-	+		
F					-	+	
G						-	+
H							-

Figure 10 Plus and Min matrix

For correlations $\rho_{i;j}$ unequal to zero, the local correlation types are determined as follows:

- In an image of the failure mechanism of the structure, assign for each layer boundary from the Plus and Min matrix a + side and a – side. See Figure 4;
- Determine in accordance with Figure 8 the before mentioned refererence axis between the two layer boundaries;
- If both sides of the reference axis have an equal sign, it is Type A. If not, it is Type B.

5.3 Weight factors

The geometrical contributions to the failure mechanism of the soil layers (or clusters) are expressed in weight factors w(i).

In the PMMS the weight factors are determined from the surface cross sections of the soil layers.

If layer boundaries are uncertain, the weight factors become distribution functions.

In that case they are generated by a 2000 draws Monte Carlo simulation from the distribution functions of the layer boundaries.

5.4 Univariate & Multivariate distributions

Using math falling outside the scope of this article, the SUM algorithm converts the surface cross sections, uncertainties in the layer boundaries, mutual correlations of w(i) and correlation types into a Model Covariance Matrix Cov($\overline{w}$) or simply W.

Then, with a Monte Carlo simulation, a 2000 draws univariate distribution is generated for each separate weight factor. In this procedure the mean values of the sum of the weight factors are set to 1.0.
Thereafter through a Cholesky decomposition of W, the 2000 draws univariate distributions of w(i) are converted into a 2000 draws multivariate distribution $\overline{w}$.

The SUM algorithm contains sub-algorithms to ensure that nowhere in both the univariate distributions w(i) and the associated multivariate distribution $\overline{w}$, weight factors less than 0,0 or greater than 1.0 are generated.
This however in such a way that the Covariance Matrix W’ that can be generated from the 2000 draws multivariate distribution $\overline{w}$, still very accurately matches the Model Covariance Matrix W from which they are generated.
This procedures may result in non-Gaussian distributions of $\overline{w}$ being generated, which resembles situations that can occur in practical cases.

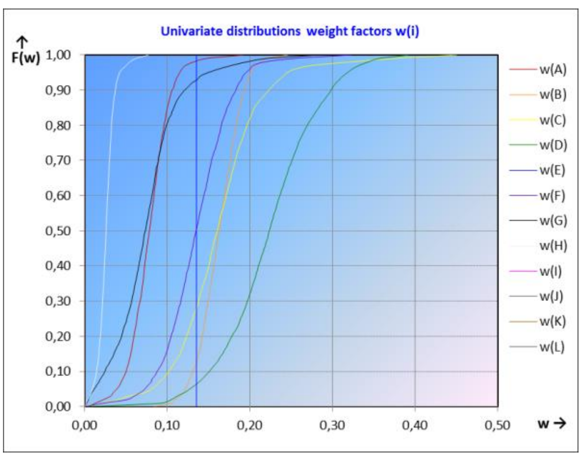

Figure 11 Univariate distributions of weight factors

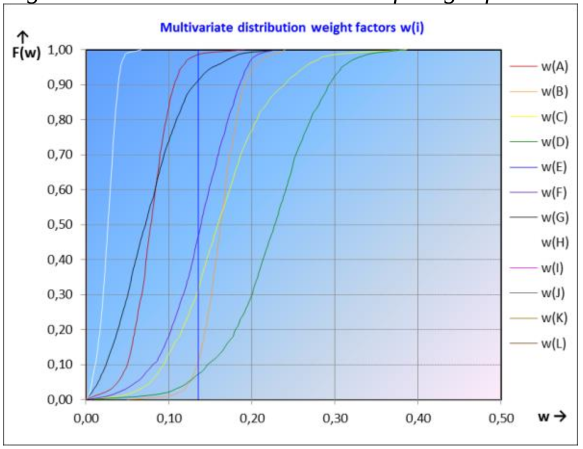

Figure 12 Multivariate distribution of weight factors

Figure 11 and Figure 12 shows univariate distributions w(i) and the associated multivariate distribution $\overline{w}$. In a graph they often look similar, but mathematically they are different.

6 LIMIT STATE FUNCTION AND RELIABILITY INDEX

With *j* denotes the number of the associated Monte Carlo draw, the limit state function Z_j in the PMMS is defined as follows:

$$Z_j = E_j - E_{c,j} \tag{6}$$

where E_j is proportionate to the incremental energy dissipated across the failure mechanism determined with the in situ actual shear strength and $E_{c;j}$ is proportionate to the incremental energy

that corresponds to the shear strength at failure. Herein is E_j a stochastic variable.
The reliability index β_j for Monte Carlo draw j is determined from:

$$\beta_j = \frac{\mu(Z_j)}{\sigma(Z_j)} = \frac{\mu(E_j) - E_{c,j}}{\sigma(E_j)} \tag{7}$$

in which $\mu(E_j)$, $\sigma(E_j)$ and $E_{c;j}$ respectively, are proportionate to the mean value, standard deviation and value at failure of the dissipated incremental energy E_j and are calculated using the following equations:

$$\mu(E_j) = \bar{w}^T \{\bar{\gamma} \circ \mu(\bar{\tau})\} \tag{8}$$

$$\sigma(E_j) = \sqrt{\{\bar{w} \circ \bar{\gamma} \circ \sigma(\bar{\tau})\}^T \, Cor(\tau) \, \{\bar{w} \circ \bar{\gamma} \circ \sigma(\bar{\tau})\}} \tag{9}$$

$$E_{c;j} = \bar{w}^T \{\bar{\gamma} \circ \bar{\tau}_c\} \tag{10}$$

Where in this case $\bar{w}$ is the associated Monte Carlo generated multivariate draw j, $\bar{\gamma}$ are the incremental strains in the failure mechanism for this draw, $\mu(\bar{\tau})$ are the mean values of the shear strengths, $\sigma(\bar{\tau})$ are the standard deviations of the shear strengths, $\bar{\tau}_c$ are the mobilized shear strengths at failure and $Cor(\tau)$ is the shear strength Correlation Matrix. The operator ○ denotes the Hadamard product of matrices.
The probability of failure $P_{f;j}$ then follows from:

$$P_{f,j} = \Phi(-\beta_j) \tag{11}$$

Where Φ is the cumulative normal distribution function.
Under the assumption that the location and shape of the failure mechanism are not very sensitive for the distribution of the weight factors, the reliability index β_s for the whole structure is calculated by averaging the failure probabilities of all Monte Carlo simulations, which is expressed by:

$$\beta_s = -\,\Phi^{-1}\left(\frac{1}{n}\sum_{j=1}^{n} \Phi(-\beta_j)\right) \tag{12}$$

Where n is the number of Monte Carlo draws and Φ^{-1} the inverse of the cumulative normal distribution function.

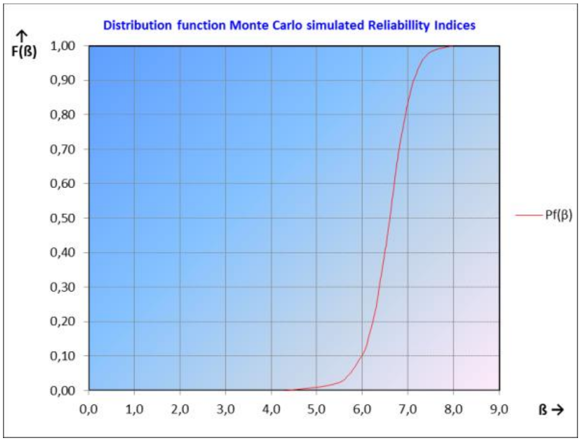

Figure 13 Distribution M.C. generated Reliability Indices

7 EXAMPLE

The effect in the PMMS of the SUM algorithm on the Reliability Index is calculated using the simple dike geometry in Figure 4.
For this geometry a Safety Factor Σ-MSF of 1.54 is calculated with PLAXIS.

Figure 14 shows the partly from PLAXIS obtained shear strength- and strain parameters as used in the PMMS of this dike geometry.
The geometrical data and its uncertainties are shown in previous paragraphs. For the layer combinations (B-C); (D-E); (D-F) and (E-F) it is assumed that these are fully correlated (ρ = 1,0).

A PMMS calculation based on these strength parameters and geometrical data resulted in a distribution of the Reliability Indices as calculated from the 2000 different Monte Carlo draws of $\bar{w}$ as shown in Figure 13 and a Reliability Index for the construction according to equation (12) of ß = 5.49. For the same case assumed that the layer boundaries are not subject to variations a Reliability Index of ß = 6.69 is calculated.

It is stated that in this example the possible influence of the variations in the locations of the layer boundaries to the shear strengths of the soil layers and to the location of the failure mechanism are not taken into account.

This is an important point for further research.

Basic information		Volume stresses		Volume strain	Mohr-Coulomb Model & Coulomb Friction Model						Largest Principal Effective Stress	SHANSEP Shearstrength f(S;OCR;m) Lognormal								
Layer	Soil type				Value in calculation		Mean value		Coefficient of Variation			Value in calculation			Mean value			Coefficient of Variation		
-	-	$\sigma'_{1;c}$	$\sigma'_{3;c}$	γ(A)	C_a	Phi_a	μ(C)	μ(Phi)	Cov(C)	Cov(Phi)	$\sigma'_{1;o}$	$S_{;a}$	$\sigma'_{1;max;a}$	$m_{;a}$	μ(S)	$\mu(\sigma'_{1;y})$	μ(m)	Cov(S)	$Cov(\sigma'_{1;y})$	Cov(m)
A	Klei, Dijks, Gedraineerd	14,69	4,17	1,00	0,00	34,00	0,00	34,00	0,000	0,065	-	-	-	-	-	-	-	-	-	-
B	Klei, Dijks, Ongedraineerd	44,80	-	1,00	-	-	-	-	-	-	44,80	0,350	74,79	0,910	0,350	74,79	0,910	0,057	0,220	0,022
C	Klei, Dijks, Ongedraineerd	57,38	-	1,00	-	-	-	-	-	-	57,38	0,350	87,37	0,910	0,350	87,37	0,910	0,057	0,220	0,022
D	Hollandveen O	62,12	-	1,00	-	-	-	-	-	-	62,12	0,390	73,13	0,850	0,390	73,13	0,850	0,051	0,230	0,024
E	Hollandveen N	41,50	-	1,00	-	-	-	-	-	-	41,50	0,390	52,50	0,850	0,390	52,50	0,850	0,051	0,230	0,024
F	Hollandveen N	24,79	-	1,00	-	-	-	-	-	-	24,79	0,390	35,79	0,850	0,390	35,79	0,850	0,051	0,230	0,024
G	Klei, Antro., Ongedraineerd	15,74	-	1,00	-	-	-	-	-	-	15,74	0,350	37,74	0,910	0,350	37,74	0,910	0,057	0,160	0,022
H	Klei, Antro., Gedraineerd	4,85	12,74	1,00	0,00	34,00	0,00	34,00	0,000	0,065	-	-	-	-	-	-	-	-	-	-

Figure 14 Strength & strain parameters

8 VERIFICATION

Simanjuntak et al. 2019 describes a successful verification for a number of cases in which the layer boundaries and the failure mechanism are fixed. Verifications for cases with uncertain layer boundaries and an uncertain failure mechanism are in progress.

9 CONCLUSIONS

This paper shows that the Probabilistic Model Macro Stability PMMS is a novel and powerful mathematical method to determine the Reliability Index β with only a limited number of FEM calculations.

The most innovative feature of the PMMS is the Stochastic Underground Model SUM, which makes it possible to also include uncertainties in the soil stratification of the geotechnical unit in determining the Reliability Index. The discussed example shows that uncertainties in the location of the layer boundaries may have a considerable influence on the calculated Reliability Index.

The general engineering approach is to use a conservative schematisation of the subsurface and not vary the ground layer boundaries.

This may lead to a too conservative Reliability Index. The PMMS enables engineers to include uncertainties in both shear strengths and in soil stratification in a simple and practical way. This may contribute to a more optimal design.

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