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Analyses of the design and stability of the fractal retaining walls built by the Inkas of Peru

Analyses de la conception et de la stabilité des murs de soutènement en fractals construits par l´Inkas du Pérou

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ABSTRACT: The Inka Empire consisted of some 10 million inhabitants. The Inkas built retaining walls for their agricultural terraces. These terraces were located on steep rock slopes. This study involves: (a) the type of design used by the Inkas in their retaining walls, and (b) a stability analysis of the retaining walls with respect to static loads. The Inca walls were made by piling up rock pieces made of granite. The size of these rocks varied between 10 cm and 2 meters. The rock size distribution used in the walls' design was fractal in nature. That is, the walls were made of a mixture of large, medium and small rock pieces. The source of the stability of the walls rested on the gravity-induced frictional resistance developed between the many contacts between the large and moderate sized rock pieces and the small rock pieces. The total frictional resistance between the rock pieces forming the walls was found to be greater than the pressures exerted by the soil behind the walls. Also, since no cement was used in the building of the walls, these are freely drained. Thus, water did not contribute to the instability of the walls. The Inca retaining walls have been stable for more than 500 years. Given this remarkable achievement in sustainable engineering, it is undoubtedly important to investigate the reason for their long-term stability. The stability and sustainability analyses of the Inca walls were made using field, theoretical and numerical (DEM) investigations.

RÉSUMÉ: L'empire Inka comptait environ 10 millions d'habitants. Les Inkas ont construit des murs de soutènement pour leurs terrasses agricoles. Ces terrasses étaient situées sur des pentes rocheuses abruptes. Cette étude porte sur: a) le type de conception utilisé par les Inkas dans leurs murs de soutènement; et b) une analyse de la stabilité des murs de soutènement vis-à-vis des charges statiques. Les murs incas ont été construits en empilant des morceaux de roche en granit. La taille de ces roches variait entre 10 cm et 2 mètres. La distribution de la taille de la roche utilisée dans la conception des murs était de nature fractale. C'est-à-dire que les murs étaient constitués d'un mélange de gros, moyens et petits morceaux de roche. La stabilité des murs repose sur la résistance de friction induite par la gravité développée entre les nombreux contacts entre les blocs de pierre de taille moyenne et grande et de petite taille. La résistance de friction totale entre les morceaux de roche formant les murs s'est avérée supérieure aux pressions exercées par le sol derrière les murs. De plus, aucun ciment n’ayant été utilisé dans la construction des murs, ceux-ci sont drainés librement. Ainsi, l'eau n'a pas contribué à l'instabilité des murs. Les murs de soutènement incas sont stables depuis plus de 500 ans. Compte tenu de cette réalisation remarquable en matière d'ingénierie durable, il est sans aucun doute important de rechercher les raisons de leur stabilité à long terme. Les analyses de stabilité et de durabilité des murs incas ont été effectuées à l'aide d'enquêtes de terrain, théoriques et numériques (DEM).

Keywords: Retaining walls; design; fractals; stability; Inkas.

1 INTRODUCTION

Pre-Columbian civilizations are sometimes thought to be primitive, inferior societies in comparison to modern ones. Some of the great achievements of pre-Columbian societies have even been credited to extra-terrestrial beings because they appear to be too advanced for such “primitive” cultures to construct (Von Daniken, 1968). The ancient structures built by pre-Columbian civilizations have outlasted many of our modern structures without the benefit of our rigorous mathematical design methods and modern building codes. The Inkas of Peru managed to build retaining walls for their agricultural terraces located at Machu Picchu. Examples of these retaining walls are shown in Figures 1 and 2.

Figure 1. Retaining wall 1 at Macchu Pichu

They also built walls in Cusco, the capital of the I inka empire, as dividing walls or as part of houses (Fig. 3). The walls shown in Figs. 1 and 2 have endured centuries of weathering, and various other naturally degenerative processes

Figure 2. Retaining wall 2 at Macchu Pichu

Figure 3. Dividing wall 3 at Cusco

on steep mountainsides with little to no maintenance. Over five hundred years after construction, the walls are still standing (Kendall, 2005). Understanding the keys to the longevity of the Inkan retaining walls will recognize the intelligence of pre-Columbian societies. If one analyze the form of the retaining walls forming part of the terraces at Machu Picchu and the dividing wall in Cusco one finds that the retaining walls at Machu Picchu (Figs. 1 and 2) are made of a few large stones, many intermediate in size stones, and a large amount of small stones. This particle size distribution of the stones is a fractal one. However, the dividing wall was made of a uniform particle size distribution (non-fractal particle size distribution)(Fig.3).

The objective of this study is to report the results of an investigation on the factors that have helped the retaining walls at Machu Picchu, to endure for more than five hundred years and explain the reasons the Inka engineers built walls that were fractal in their particle size distribution for the Machu Picchu agricultural terraces, and non-fractal for the walls used as a dividing devices or as part of houses.

2 FRACTAL ANALYSIS OF THE WALLS

Many of the retaining walls at Machu Picchu have a peculiar size distribution. The walls are made of a mixture of rock pieces of different sizes that makes the distribution to be fractal in nature. Figures 1 and 2 show examples of walls at Machu Picchu. Figures 4 and 5 represent an AutoCAD (2007) plot of the profiles of different stones forming part of the walls shown in Figures 1 and 2. The plots shown in Figures 4 and 5 are also used to provide the areas and perimeters of the stones. In order to obtain the areas and perimeters for the plots shown in Figures 4 and 5, the average high (2.5 meters) of the walls was used as a scale.

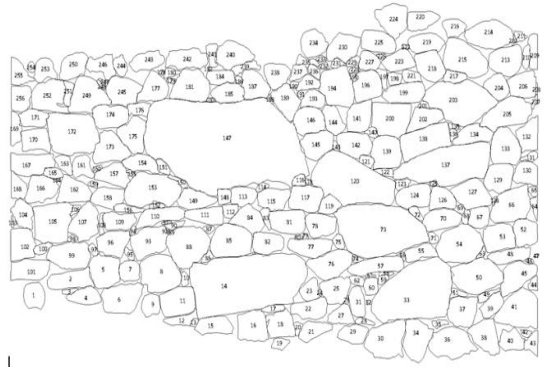

Figure 4. AutoCAD plot of wall 1

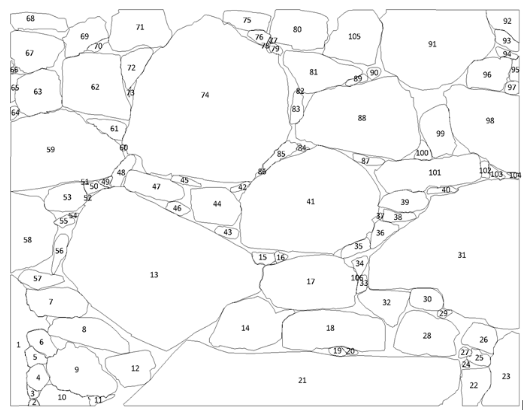

Figure 5. AutoCAD plot of wall 2

According to Tyler and Wheatcraft (1992), and Hyslip and Vallejo (1997) a distribution of particles by size can be considered fractal in nature if the following relationship is followed:

$$\mathrm{N}(A > a) = ka^{-D_F/2} \qquad (1)$$

where N(A>a) is the total number of particles with an area A greater than a given area a, and

D_F is the fractal dimension of the stone size distribution. If one plots in a log-log paper N(A>a) versus the values of a obtained from Figs. 4 and 5 , one can obtain the fractal dimension (D_F) of the size distribution of the stones forming part of Figures 1 and 2. This has been done in Figs. 6 and 7.

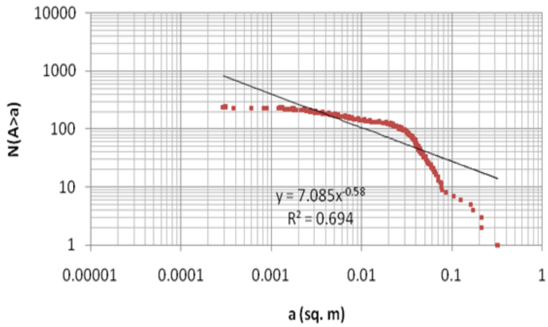

Figure 6. Plot to calulate fractal dimension of wall 1

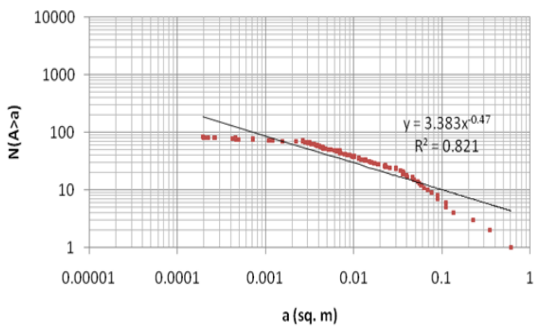

Figure 7. Plot to calulate fractal dimension of wall 2

The value of the power in the equations shown in Figs. 6 and 7 is equal to $D_F/2$. Using the value of the power we can obtain the fractal dimension of the particle size distributions of the walls 1, and 2 . The values of D_F are shown in Table 1.

According to Tyler and Wheatcraft (1992), a particle size distribution (PSD) with D_F between 0 and 3 represents a PSD that is controlled by large particles. This is in fact the case if one examines Figs. 1 and 2.

Table 1. Values of D_F for walls in Figs. 1and 2.

Wall No.	**$D_F/2$**	**D_F**
1	0.58	1.16
2	0.47	0.94

3 BENEFITS OF THE FRACTAL DESIGN FOR THE WALLS

The Discrete Element Method (DEM) will be used next in order to gain an understanding why the Inka engineers designed their walls used as retaining structures using stones with a size distribution that was fractal in nature (Figs. 1 and 2) (Itasca, 2002).

To accomplish this, a container 60 cm in height, 40 cm in width, and 1 cm in thickness was filled with 684 disks of different dimensions making a mixture that had a fractal particle size distribution D_F = 2.925. Fig. 8 shows the arrangement of the disks in the mixture, and Fig. 9 shows the force chains induced by gravity in the mixture of disks. The properties of the disks used in the DEM study are shown in Table 2.

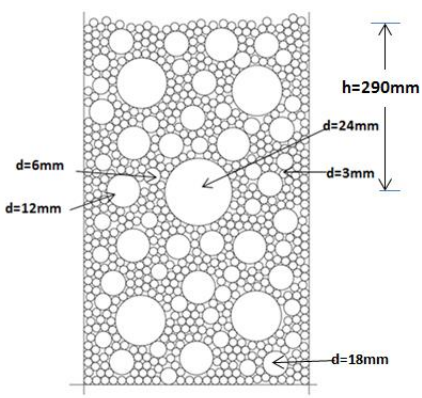

Figure 8 .Distribution of disks in the container with 60 cm in height, 40 cm in width, and 1 cm in thickness.

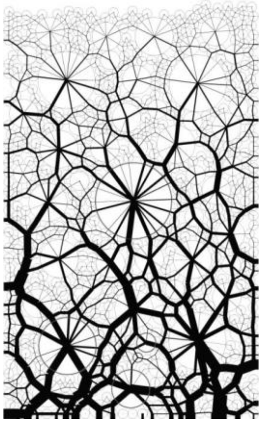

Figure 9. Gravity induced force chains in the disks

Table 2 Properies of disks used in DEM

Property	Value
Shear stiffnes, k_s	$3x10^5$ kN/m^2
Normal stiffnesm k_n	$9x10^6$ kN/m^2
Friction between disk	35^o
Density of disks (ρ)	2700 kg/m^3
Gravity acceleration	9.81 m/sec^2

3.1 *Gravity force values induced in the disks located at the same depth*

The value of the gravity forces induced on the largest particle located at a depth h=290 mm by neighbouring disks is shown in Fig. 10 and Tables 3. The value of each of the forces exerted by neighbouring disks on the selected disk (d = 0.024 m) which is located at h = 290 mm (Fig. 8) is provided by the DEM software (Itasca, 2002). The same type of analysis conducted on the largest disk with d = 24 mm and located at depth h = 290 mm was conducted on the disks with d = 12 mm, d = 6 mm, and d = 3 mm (Figs. 8 and 9). The results are shown in Table 4.

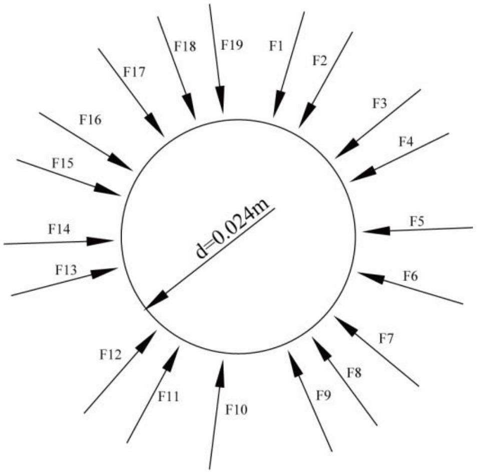

Figure 10. Gravity induced forces by neighbouring disks on the disk with d = 0.024 meters using Fig. 9

Table 3 Gravity forces on particle with d= 24mm

Neighbor disk number	Force value , F_n (N)
1	6.68
2	5.46
3	6.06
4	2.04
5	2.20
6	4.72
7	5.31
8	3.16
9	11.19
10	17.95
11	5.55
12	0.59
13	4.93
14	0.44
15	0.50
16	8.05
17	12.34
18	0.44
19	0.48

Total normal gravity force, F_n: 98.09 N

Table 4. Total gravity normal force acting on disks with d = 12, 6, and 3 mm

Disk diameter (mm)	Number of neighbors	Total normal gravity force, F_n (N)
12	8	88.85
6	4	31.28
3	3	8.60

An analysis of Tables 3 and 4 indicates that as the disks become smaller, the number of normal forces on the disks decreased in number and in value because the number of contacts with neighbouring disks decreased in value. Thus, a wall with a fractal mixture of stones will develop higher frictional resistance (more contacts) than walls constructed by a uniform size distribution (small number of contacts).

3.2 Resisting and lateral forces acting on particles located at the same depth

The resisting force (F_r) offered by each disk is equal to the product of the total normal gravity force (F_n, from Tables 3 and 4)) to the disk times the coefficient of friction between the particles (tan δ = tan 35° = 0.7) (Table 2). In an equation form,

$$F_r = F_n \tan \delta \qquad (2)$$

If the fractal mixture shown in Figs. 11 and 12 form part of a gravity retaining wall, the soil behind the wall will exert a lateral earth pressure at rest that can be obtained from Rankine theory assuming the wall to be smooth. The material behind the retaining walls at Machu Picchu is a granular type of material with a friction angle $\phi = 40^o$ and a unit weight, $\gamma = 16$ kN/m^3 (Fontanese, 2010). Using Rankine's theory, the lateral at rest pressure in the wall at a depth, h, is (Das, 1984):

$$\sigma_h = \gamma h (1 - \sin\phi) \qquad (3)$$

To obtain the lateral force, F_h, at depth h = 0.29 meters (Fig. 8), one can obtain first the lateral pressure at this depth h using Eq. (3). This pressure will represent the average lateral pressure that acts on the area of the disks with diameters d = 0.024 m, d = 0.012 m, d = 0.006 m, and d =0.003 m forming the wall shown in Fig. 8. Therefore the lateral force, F_h, on each of these disks at the same depth h can be obtained from:

$$F_h = [(\pi d^2/4)][\sigma_h] \qquad (4)$$

Table 5 shows the values of the resisting force, F_r, and the values of the lateral force, F_h, for the case of the four disks located a h = 0.29 m (Fig. 8).

Table 5. Values of the resisting and lateral forces

Disk diameter (m)	F_r (N)	F_h (N)
0.024	68.66	0.75
0.012	62.20	0.19
0.006	21.90	0.047
0.003	6.02	0.012

An analysis of Table 5 indicates that the disks forming the wall of Figs. 8 and 9 are stable with respect to their removal by the lateral forces ($F_r > F_h$). That is, the disks (particles) will not be removed by the lateral force exerted on each of the disks by the granular material located behind the wall (backfill). This seems to explain the stability of the stones forming the free draining walls located at Machu Picchu and shown in Figs. 1 and 2. These walls were free draining because the Inkas did not have cement anD the water can drain easily through the open spaces between the stones (Wright and Valencia-Zegarra, 2000). The stability of the stones forming the walls was the result of gravity induced frictional restistance (F_r) developed when the stones were placed one on top of the other. In a previous study conducted by Vallejo and Fontanese (2014), the walls shown in Figs. 1 and 2 were also found to be stable with respect to sliding and overturning.

4 CONCLUSIONS

From this study the following conclussions can be reached:

(1) The walls forming part of the agricultural terraces at Machu Picchu in Peru are made of a stone size distribution that is fractal in nature. That is, the walls are made of a few very large in size, many moderate in size, and multiple small in size stones.

(2) This fractal distribution causes the enhancement of the gravity induced frictional resistance between the stones forming the wall. This frictional resistance is the highest for the big stones and decreases in value as the size of the stones decreased.

(3) The frictional resistance between the stones prevented their removal by the lateral force induced on the wall's stones by the backfill.

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