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Proposed Analytical and Numerical Methods to Assess Tunnel Excavation Stability

Méthodes Analytiques et Numériques Proposées pour Evaluer la Stabilité des Excavations dans les Tunnels

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ABSTRACT: Tunnel collapses are often reported to occur during excavations or shortly afterwards, yet excavation stability analyses are seldom included into tunnel design documents. Available tunnel stability methods are briefly discussed, focusing on their main limitations, such as homogeneous dry soil conditions only, thus not allowing for groundwater flow seepage forces and a layered soil profile. A proposition is made on two distinct stability procedures, each recommended to be applied at different design stages:

1. A simple analytical formulation known as Caquot's model, but modified to allow for groundwater flow and multiple materials in the soil mass, and extended to simulate three dimensional geometries.
2. A three dimensional numerical model, such as a Finite Element Method.

The basic assumptions and limitations of both procedures are discussed, followed by comparative evaluations against laboratory physical models. It is concluded that the straightforward analytical model is a convenient stability evaluation tool; however due to a series of limitations its use should be restricted to preliminary design analyses. A three dimensional numerical method is able to overcome analytical model limitations with improved reliability, being recommended to be deployed at a detailed design stage.

RÉSUMÉ: Tunnel s'effondre on rapporte souvent à se produire au cours de fouilles ou peu de temps après, mais les analyses de stabilité excavation sont rarement inclus dans les descriptifs de tunnel. Méthodes de stabilité de tunnel disponible discute brièvement, en se concentrant sur leurs principales limites, comme seule condition de sol sec homogène, ne permettant ne pas aux forces de suintement flux des eaux souterraines et un profil de sol stratifié. Une proposition est faite sur deux procédures distinctes de stabilité, chacun a recommandé à appliquer aux stades de la conception différente :

1. Une formulation analytique simple connu comme modèle de Caquot, mais modifié pour tenir compte de l'écoulement des eaux souterraines et des matériaux multiples dans la masse de sol et étendu pour simuler trois géométries dimensionnelles.
2. Trois dimension modèle numérique, comme une méthode d'éléments finis.

Les hypothèses de base et les limites des deux procédures sont discutés, suivie des évaluations comparatives contre modèles physique de laboratoire. On en conclut que le modèle analytique simple est un outil d'évaluation de stabilité pratique; toutefois, en raison d'une série de limitations, son utilisation devrait être restreinte à des analyses de conception préliminaire. Trois dimensions méthodes numériques est en mesure de surmonter les limitations du modèle analytique avec une fiabilité améliorée, étant recommandée pour être déployé à un stade de la conception détaillée.

Keywords: Tunnels; Excavation; Stability; Design Criteria.

1 INTRODUCTION

Tunnel excavation accidents often result in serious consequences, including loss of life and major disruptions at ground level. However it is felt that there is no agreement upon a stability method to be used by tunnel designers in order to mitigate their occurrence. It should be noted that tunnel stability methods are absent from most National Codes. Consequently, excavation stability calculations are rarely seen on tunnel design documents.

The objective of this paper is to propose two distinct stability methods to be considered for tunnel excavation stability. One is a simple but limited analytical method, the other is an advanced three dimensional numerical method.

2 BRIEF OVERVIEW ON STABILITY METHOD SHORTCOMINGS

The lack of a widely recognised tool for tunnel's stability is in sharp contrast to what is practiced in other geotechnical fields, such as slope stability. Slope stability methods are well established and included in National Code annexes or official publications, such as Eurocode 7 "Geotechnical Design Worked Examples", Bond et al. (2013), or ASCE "Slope Stability Analysis by the Limit Equilibrium Method", Huang (2014).

In contrast to Slope Stability, tunnel stability methods are not covered in most National Codes. Various different calculation procedures can be found in bibliography. However, these procedures are generally restricted to specific conditions such as homogeneous and dry soil, not allowing for groundwater flow destabilizing seepage forces. Also these methods may be limited to a certain failure mode, e.g. tunnel face failure only, not including an unsupported tunnel length which is a common feature in conventionally excavated tunnels.

Stability results may vary considerably depending on the adopted methodology. As an example, many published methods are based on Horn's (1961) limit equilibrium mechanism and will give wide scattered results, depending on the transversal stress assumption at sliding wedges. The graph of Fig. 1, shown by Kirsch (2008), illustrates the discrepancy on normalised face support pressures $(p/\gamma D)$ at failure, within Horn's mechanism, and between other methods.

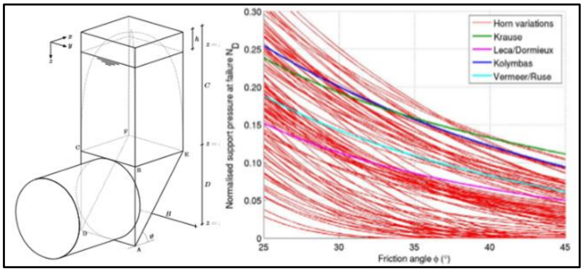

Figure 1. Horn's mechanism, left, and discrepant support pressure results from various Authors, as shown by Kirsch (2008).

3 REQUIRED ATTRIBUTES OF A TUNNEL STABILITY METHODOLOGY

A general excavation stability solution for tunnels should be applicable to failure modes and geotechnical conditions often encountered in practice, and listed herewith:

- *Collapse and Blow Out modes of failure.* Collapse is the most frequent failure mode. However, blow out failures associated to excess slurry pressure in closed shields should also be considered.
- *Variable unsupported excavation length.* Failure mechanism should not be restricted to tunnel face, but extended to a given distance behind it, such as distance to invert arch in conventional tunnels, (three dimensional model - 3D), and extending to the whole tunnel length as in the case where there is no invert arch (two dimensional model - 2D).
- *Multi layered soil profile.* Layers of different materials should be allowed in the model, as opposed to a single homogeneous material.
- *Drained and Undrained material behaviour.* A given multi layered soil profile should enable inclusion of undrained clays and drained sands, coalescing in a unique analysis.

- *Groundwater Flow.* It significantly impairs excavation stability, for both collapse (inward flow to excavation, such as tunnels at atmospheric conditions below groundwater level), and blow out (outward flow from shallow slurry shields at excessive pressure). Therefore, flow net seepage forces must be included in this model.

4 PROPOSED STABILITY METHODOLOGY

Two distinct methods are proposed herein:

- *Analytical Model.* Also denominated as Thick Wall Model, since its equilibrium configuration is based on either a Thick Wall Cylinder (TWC - 2D analysis) or a Thick Wall Sphere (TWS - 3D analysis). It is derived from the simple formulation developed by Caquot (1934), but modified to allow for 2D and 3D analyses, drained and undrained behaviour, multiple materials in the soil mass, and groundwater flow. This model permits fast and effortless assessments of excavation stability conditions. However, due to its simplifying limitations, it should be considered as a preliminary tool suited to early stages of a tunnel design.
- *Three Dimensional (3D) Numerical Methods.* Models such as the Finite Element Method are complete solutions that obey equilibrium, strength criterion, stress strain behaviour and strain displacement (compatibility) conditions. These methods usually demand human and computational resources compatible with an advanced design stage, and enable complementation of the preliminary analytical assessments with improved accuracy.

5 ANALYTICAL MODEL – BASIC EQUATIONS

The analytical model is based on Thick Wall Cylinder (TWC) or Thick Wall Sphere (TWS) geometries (2D or 3D configurations respectively). The basic concepts of this model theory have been dealt with by several authors, since its inception by Caquot (1934), and followed by further developments such as published by Jaeger & Cook (1975), Davis et al. (1980), Mühlhaus (1985), Carranza Torres et al. (2013), and Sozio (2016, 2017).

The models presented in this paper are based on Caquot's original model, with the following modifications:

- Groundwater flow seepage forces are included into the equilibrium equation.
- Multiple materials are accounted for, as concentric multiple cylinders or spheres.
- Two Dimensional (2D) and Three Dimensional (3D) solutions are developed by expressing equilibrium equation in cylindrical or spherical coordinates, respectively.

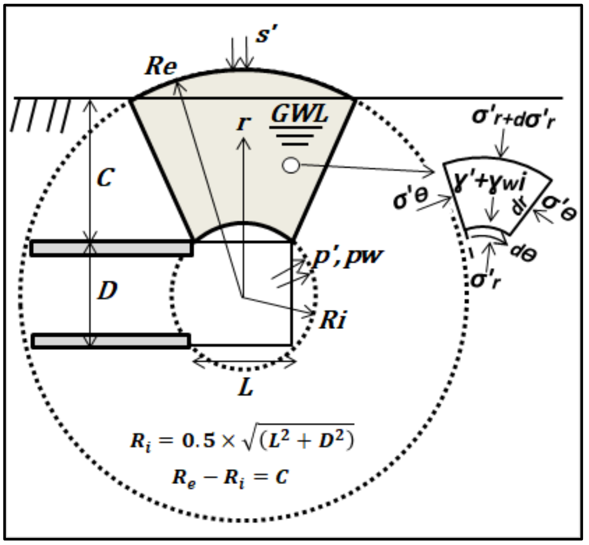

Figure 2. Cross section view of a Thick Wall single material model.

The complete theoretical development of Thick Wall Model equations is presented by Sozio (2016, 2017), including collapse and blow out modes of failure, drained and undrained soil behaviour, in two or three dimensions. A basic geometrical arrangement applicable to a Thick Wall Model is shown at Fig. 2, and its fundamental equations are described subsequently.

5.1 Equilibrium equation in cylindrical or spherical coordinates.

$$\frac{d\sigma'_r}{dr} + (n-1)\frac{(\sigma'_r - \sigma'_\theta)}{r} = -\gamma' - \gamma_w \times i \qquad (1)$$

$\sigma'_r, \sigma'_\theta$: effective radial and tangential stresses, respectively.
$n = 2$, 3: cylindrical or spherical coordinates for 2D or 3D analyses respectively.
γ': natural unit weight if above groundwater level (GWL), or submerged unit weight if below GWL.
γ_w: fluid unit weight, usually water, 10kN/m³.
i : hydraulic gradient. The term $\gamma_w \times i$ is known as (volumetric) seepage force.
It is implicit from Eq. (1) that equilibrium of a soil element is based on submerged unit weight (when below GWL) and seepage forces, refer to Lambe & Whitman – Chapter 17 (1969). However, seepage forces do not need to be considered for undrained analyses, as these are solely based on total stresses.

5.2 *Mohr Coulomb strength criterion.*

5.2.1 *Drained behaviour*

Drained intrinsic parameters, effective cohesion (c') and effective friction angle (ϕ').

$$\sigma'_1 = \sigma'_c + \sigma'_3 \times \lambda \tag{2}$$

$$\lambda = \frac{1+sin\phi\prime}{1-sin\phi\prime} \tag{3}$$

$$\sigma'_c = \frac{2\times c\prime\times cos\phi\prime}{1-sin\phi\prime} \tag{4}$$

σ'_1 and σ'_3 are respectively the major and minor principal stresses.

5.2.2 *Undrained behaviour*

Undrained shear strength including allowance for a linear shear strength increase with depth (z), from ground level.

$$S_u = C_k + C_z \times z \tag{5}$$

5.3 *Darcy law for groundwater flow*

Flow through a surface of a cylinder (Q_{2D}), or through surface of a sphere (Q_{3D}).

$$Q_{2D} = k \times \frac{dh}{dr} \times 2\pi r \tag{6}$$

$$Q_{3D} = k \times \frac{dh}{dr} \times 4\pi r^2 \tag{7}$$

k is soil isotropic permeability and $\frac{dh}{dr} = i$ is the hydraulic gradient.

The integration process leading to determination of hydraulic gradients follows the conceptual deduction presented by Taylor (1948), (Chapter 9-16 - Simple Cases of Radial Flow).

6 ANALYTICAL MODEL – INTEGRATION PROCEDURE

The detailed steps of the integration procedure leading to the final collapse or blow out tunnel pressures are shown by Sozio (2016, 2017). A brief description is shown herein, noting that groundwater flow gradients are determined first, followed by solving effective stresses through equilibrium and Mohr Coulomb equations.

6.1 *Integration of groundwater flow.*

Eq. (6) or (7) is integrated throughout the cylinder or sphere, adopting the following hydraulic head boundary conditions (refer to Fig. 2):

$$R_i + \frac{pwi}{\gamma_w} \tag{8}$$

$$R_e + \frac{pwe}{\gamma_w} \tag{9}$$

pwi and pwe are fluid pressures at internal and external boundaries. The external radius R_e of the hydraulic boundary may be coincident with groundwater level ($pwe = 0$ in this case), or with ground surface level if $pwe \geq 0$, e.g. ground surface level is at the bottom of a lake, river, sea, etc. The resulting hydraulic gradients for 2D and 3D geometries are, respectively:

$$i = \frac{A_{2D}}{r} \tag{10}$$

$$i = \frac{A_{3D}}{r^2} \tag{11}$$

The hydraulic constants, respectively in units of length and length squared, are:

$$A_{2D} = \frac{(R_e - R_i) + \left(\frac{pwe-pwi}{\gamma_w}\right)}{ln\left(\frac{R_e}{R_i}\right)} \tag{12}$$

$$A_{3D} = \frac{R_e \times R_i}{(R_e - R_i)} \times \left\{(R_e - R_i) + \left(\frac{pwe-pwi}{\gamma_w}\right)\right\} \tag{13}$$

Hydraulic equations are applicable to both inward and outward flow, which will depend on the relative values of internal and external hydraulic heads.

6.2 Introduction of hydraulic gradient into equilibrium equation.

Hydraulic gradients from Eqs. (10) and (12), or (11) and (13), are introduced into equilibrium equation (1).

6.3 Combination of equilibrium equation and Mohr Coulomb strength criterion.

Mohr Coulomb strength criterion is expressed in terms of principal stresses. For collapse mode, radial is the minor principal stress. For blow out mode, radial is the major principal stress.
The combination of equilibrium equation (hydraulic gradient included) with Mohr Coulomb criterion (rewritten in terms of radial and tangential stresses) results in a differential equation, function of $d\sigma'_r$ and dr.
This differential equation is then integrated. Boundary conditions for this integration are internal and external effective stresses, namely tunnel effective stress ("support or blow out pressure") and ground level "surcharge" stress.
It is important to distinguish the eventual two components, stress or pressure, acting at tunnel excavation boundary:

6.3.1 Fluid pressure pwi

Included in Eqs. (12) or (13), simulates a fluid pressure from slurry or earth pressure balanced shields, or eventual compressed air tunnels.

6.3.2 Effective radial stress σ'_r

It represents a mechanical stress, provided by a tunnel lining. Also, a temporary support such as a shield cylindrical steel plate is in this category; however, this is generally undesirable as a support since it brings propulsion and manoeuvring problems to shield advancement.

Thus, in practical terms, stresses acting at unlined tunnel headings are either null, or are provided by fluid pressure.

7 ANALYTICAL MODEL – MAIN EQUATIONS

The full set of analytical equations covering collapse or blow out modes of failure, 2D and 3D geometries, drained behaviour, including groundwater flow, and undrained behaviour, are presented in Sozio (2017). Of these, the most likely to be used in actual tunnel design are 3D equations for collapse and blow out, drained and undrained behaviour, shown below. Notation is as indicated at Fig. 2 and throughout the basic equations of the analytical model, Section 5.

7.1 3D Drained behaviour

7.1.1 Collapse

$$p' = \left[\left\{\frac{Ri}{Re}\right\}^{2(\lambda-1)} \times \left\{s' + \frac{\sigma'_c}{(\lambda-1)} - \frac{\gamma' Re}{2(\lambda-{}^3/_2)} - \frac{\gamma_w A_{3D}}{Re(2\lambda-1)}\right\}\right] + \left\{\frac{\gamma' Ri}{2(\lambda-{}^3/_2)} + \frac{\gamma_w A_{3D}}{Ri(2\lambda-1)} - \frac{\sigma'_c}{(\lambda-1)}\right\} \quad (14)$$

7.1.2 Blow Out

$$p' = \left[\left\{\frac{Re}{Ri}\right\}^{2(1-\frac{1}{\lambda})} \times \left\{s' + \frac{\sigma'_c}{(\lambda-1)} + \frac{\gamma' Re}{2\left({}^3/_2-\frac{1}{\lambda}\right)} + \frac{\gamma_w A_{3D}}{Re\left(1-\frac{2}{\lambda}\right)}\right\}\right] - \left\{\frac{\gamma' Ri}{2\left({}^3/_2-\frac{1}{\lambda}\right)} + \frac{\gamma_w A_{3D}}{Ri\left(1-\frac{2}{\lambda}\right)} + \frac{\sigma'_c}{(\lambda-1)}\right\} \quad (15)$$

The non-determination arising from $\lambda = 3/2$ or $\lambda = 2$ can be sorted out by adopting an infinitesimal modification to the friction angle (e.g. ∓0.001°). The above equations are not valid for $\lambda = 1$, which represents undrained behaviour.

7.2 3D Undrained behaviour

7.2.1 Collapse

$$p = s + (4C_z + \gamma) \times (R_e - R_i) - \left(4(C_k + C_z \times R_e)\right) ln\left(\frac{R_e}{R_i}\right) \quad (16)$$

7.2.2 Blow Out

$$p = s - (4C_z + \gamma) \times (R_e - R_i) + \left(4(C_k + C_z \times R_e)\right) ln\left(\frac{R_e}{R_i}\right) \quad (17)$$

It should be noted that the limiting pressure p, is balanced at tunnel crown, as can be depicted from Fig. 2. For uniform shear strength soils in collapse condition, it is recommended to balance the limiting pressure at tunnel axis in order to account for sidewall failure. This requires adding the term $\gamma D/2$ to the limiting pressure.

8 ANALYTICAL MODEL – MULTIPLE MATERIALS

A stratified multiple material soil profile can be simulated as a series of concentric spheres (3D) or cylinders (2D), the inner radius of an upper sphere (cylinder) being equal to the outer radius of the immediately below sphere (cylinder).

Two basic assumptions are required in order to establish a multiple material solution:

- Continuity of flow through the system.
- Effective radial stress compatibility at material boundaries.

Flow rate through the system depends on the relative permeability of each material, and a set of equations can be established and solved by assuming fluid pressure values at innermost and outermost boundaries.

Effective stresses are determined through a top – down procedure where the collapse (or blow out) stress for an upper material sphere (cylinder), as determined from Eqs. (14 - 17), is input as the surcharge stress at the immediately below sphere (cylinder). The above mentioned procedures are described in detail in Sozio (2017), and can be easily implemented into a spreadsheet.

9 LIMITATIONS OF THE ANALYTICAL MODEL

- Collapse and blow out equations are the result of a simplified model where body forces emulating gravity are radial, as opposed to true vertical gravity. Furthermore it can be seen from Fig. 2 that equilibrium is only applicable to a sector of sphere or cylinder, but not to the whole soil mass bounded by horizontal surface at ground level. Therefore, although its results are generally conservative compared to Finite Element Methods and centrifuge test data, it cannot be stated that this analytical model is a genuine Lower Bound solution, where limiting pressures should be always on the safe side.
- No information is provided on displacements in the soil mass, prior to failure.
- Tunnel excavation geometries generally do not match spherical or cylindrical arrangements implicit from equilibrium equation, such mismatch being a source of inaccuracy.
- Local Failure, as defined by Davis et al. (1980) is not accounted for in the analytical model.
- Geotechnical features such as mixed face conditions and inclined layers, as well as soil strengthening techniques cannot be adequately reproduced in analytical models.
- Only continuum media can be considered in the analytical model; discontinuum mechanics involving falling blocks and sliding wedges should be treated with limiting equilibrium statics, as established in the field of rock mechanics.

10 THREE DIMENSIONAL NUMERICAL METHODS

Three dimensional (3D) numerical methods such as the Finite Element Method have the potential to enable tunnel stability analyses as a most reliable procedure, being able to overcome analytical model limitations.

Being a complete solution accounting for both stress strain and strain displacement relationships, numerical methods can provide information on displacements prior to failure.

Advanced software embedded procedures such as the arc length control ensure a robust numerical approach to failure conditions.

A sufficient amount of experience and theoretical background is required from the user, and this can be considered as the most significant limitation of these methods.

11 COMPARISONS WITH PHYSICAL MODEL TESTS

11.1 Drained behaviour

Collapse limiting pressure ratios $p/\gamma D$ from centrifuge tests published by Atkinson & Potts (1977), for a plane strain tunnel in a dry cohesionless sand Ø=50°, are compared to 2D analytical model (TWC) and Plaxis (2014) results, shown at Fig. 3, and practically independent of cover to diameter ratio (C/D), for C/D>≈0.5.

Test data are slightly scattered, compared to numerical and analytical methods, with TWC being on the conservative side.

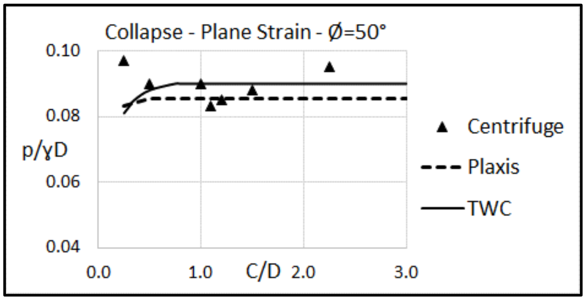

Figure 3. Dry sand laboratory test data from Atkinson & Potts (1977) compared to TWC and Plaxis.

11.2 Undrained behaviour

Centrifuge test results expressed as Stability Ratio (N), defined and published by Davis et al. (1980), are plotted at Fig. 4 and compared to 2D analytical model (TWC) and Plaxis (2014) results, for a constant undrained shear strength ratio $\gamma D/Su = 2.6$. A uniform pressure balanced at tunnel axis was considered as defined by Davis et al (1980), and as recommended at Section 7.2.

Plaxis (2014) results are in agreement with centrifuge data, and TWC is on the safe side (C/D is the cover to diameter ratio).

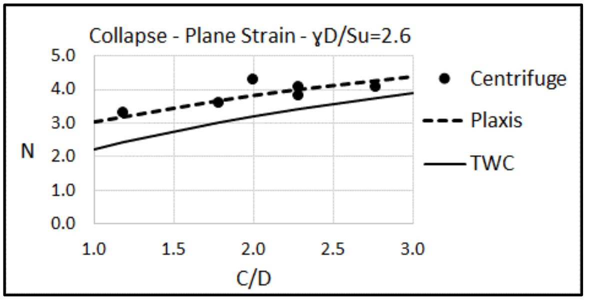

Figure 4. Stability ratio values (N) from Davis et al. (1980) compared to TWC and Plaxis.

12 POTENTIAL APPLICATIONS

Examples of 3D collapse and blow out analyses, including undrained, or drained behaviour with groundwater flow, are described in detail by Sozio (2016, 2017). A typical numerical model collapse application, Plaxis (2014), is shown at Fig. 5.

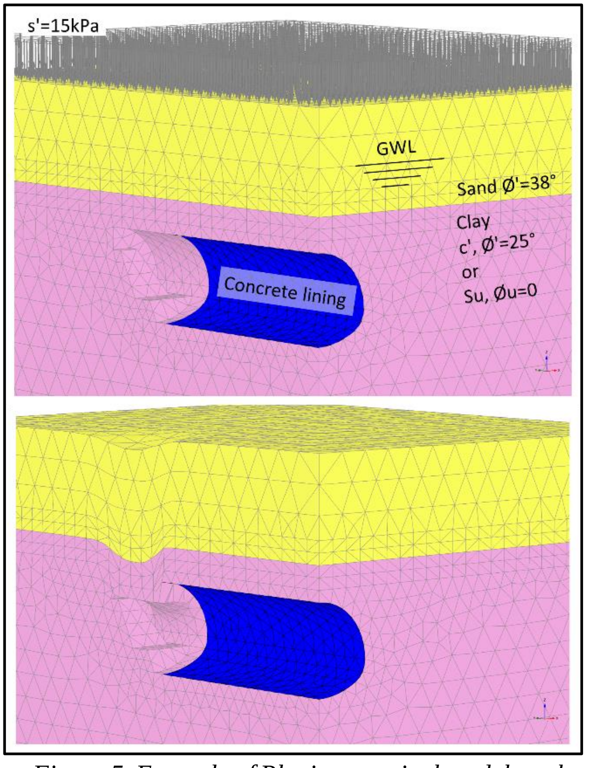

Figure 5. Example of Plaxis numerical model mesh, top, and its magnified deformed shape, bottom.

It consists of a 10m diameter sprayed concrete lined tunnel excavated in a clay layer with variable thickness above crown, beneath a sand layer and below GWL. Clay undrained or drained cohesive strengths at failure (collapse), from numerical and analytical models, are shown at Fig. 6. The analytical model (TWS) is slightly on the conservative side.

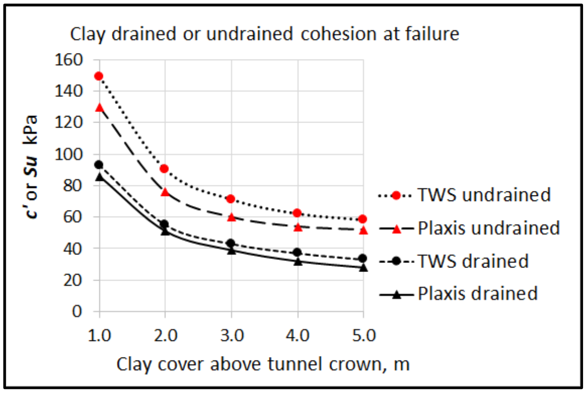

Figure 6. Minimum cohesive strengths vs. clay cover, Plaxis and TWS undrained and drained analyses.

13 CONCLUSION

As tunnel excavation accidents are continuously being reported, it is felt that a standard calculation stability methodology should be mandatory as a tunnel design criteria. A proposed methodology is described herein, based on a simple but limited analytical model, applicable to preliminary design analyses, followed by more accurate three dimensional numerical methods, appropriate to be deployed at detailed design stages.

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